1. The antipodal map $f: S^n \to S^n$ is the map given by $f(x) = -x$. Show that if $g: S^n \to S^n$ is any map such that $g(x) \neq x$ for all $x \in S^n$ then $g \simeq f$.

2. Recall that the mapping cone of a map $f: X \to Y$ is the space $C_f := X \times [0,1] \cup Y / \sim$ where $\sim$ is the equivalence relation given by $(x,1) \sim f(x)$ and $(x,0) \sim (x',0)$ for all $x,x' \in X$. Show that if $f,g: X \to Y$ are maps such that $f \simeq g$ then $C_f \simeq C_g$.

3. Show that every $3 \times 3$ matrix with positive real entries must have an eigenvalue $\lambda \in \mathbb{R}$, $\lambda > 0$. (Hint: Consider the triangle $\Delta^2 \subset \mathbb{R}^3$ defined by $\Delta^2 := \{(x,y,z) \in \mathbb{R}^3 \mid x + y + z = 1; x,y,z \geq 0\}$ Notice that $\Delta^2$ is homeomorphic to the disc $D^2$, so Brouwer fixed point theorem applies to the maps $\Delta^2 \to \Delta^2$.)

4. Let $f,g: [0,1] \to [0,1] \times [0,1]$ be paths in the square such that $f(0) = (0,a)$, $f(1) = (1,b)$, $g(0) = (c,0)$, and $g(1) = (d,1)$ where $0 \leq a,b,c,d \leq 1$. Show that $f(s) = g(t)$ for some $s,t \in [0,1]$. (Hint: use Brouwer fixed point theorem.)

5. Let $M$ be the Möbius band and let $\partial M$ denote the boundary of $M$. Show that $\partial M$ is not a retract of $M$.

6. Compute the fundamental group of the following spaces.
   a) The space $X$ obtained from $\mathbb{R}^3$ by removing $n$ straight lines intersecting at the origin.
   b) The space $Y$ obtained from $\mathbb{R}^3$ by removing the circle $S^1 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1, z = 0\}$ and the z-axis.
   c) The space $Z$ obtained from two copies of the torus $S^1 \times S^1$ by identifying the circle $S^1 \times \{x_0\}$ in one torus with the circle $\{x_0\} \times S^1$ in the other torus.
7. **(Extra Credit)** Let $S^3$ be the 3-dimensional sphere, let $A \subset S^3$ be a closed subspace of $S^3$, and let $x_0 \in S^3$. Assuming that the space $S^3 - (A \cup \{x_0\})$ is path connected show that the inclusion map

$$j: S^3 - (A \cup \{x_0\}) \hookrightarrow S^3 - A$$

induces an isomorphism of the fundamental groups.