Lecture 6

p. 128 #3, 7, 8, 10, 18, 35, 38

Review

Def $f(x)$ is <u>continuous at a</u> if $\lim_{x \to a} f(x) = f(a)$.

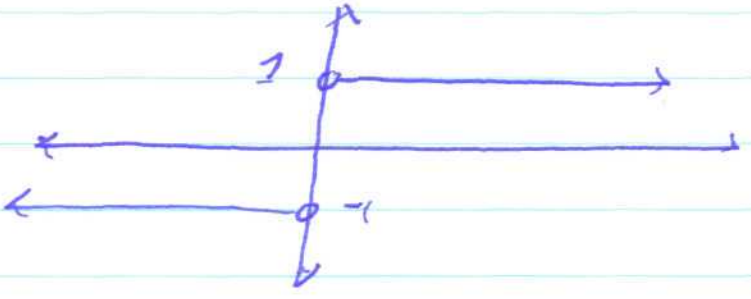
Otherwise f is <u>discontinuous</u> at a.

Remark Must have $\lim_{x \to a} f(x)$ exists, $f(a)$ defined and they are =.

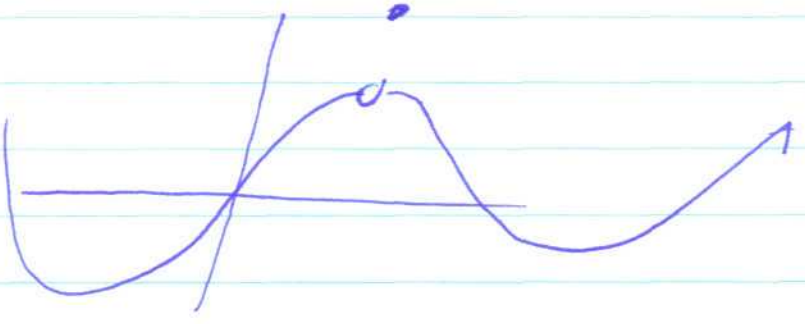
<u>Types of discontinuities</u>

1. If $\lim_{x \to a^+} f(x)$, $\lim_{x \to a^-} f(x)$ exist but are $\neq$ <u>jump</u>

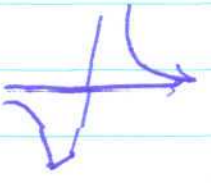
Ex $f(x) = \frac{x}{|x|}$ has a <u>jump discontinuity</u> at $x = 0$



2 If $\lim_{x \to a} f(x)$ exists but is not $f(a)$, <u>removable</u> discontin



3. $f(x) = \frac{1}{x}$ has an <u>infinite discontinuity</u> at $x = 0$.



Def f is <u>continuous from the right</u> at $x=a$ if $\lim_{x\to a^+} f(x) = f(a)$.

continuous from the left at $x=a$ if $\lim_{x\to a^-} f(x) = f(a)$.

Ex $f(x)=\sqrt{x-1}$ at $x=1$ is cont from right.

Def $f(x)$ is continuous on an interval I, if cont at every point in I, one sided on end.

$f(x)=\sqrt{x-1}$ is continuous on $\sqrt{x-1}$.

$f(t)=\sqrt{t-3}+\sqrt{8-t}$ continuous on $[5,8]$.

<u>Examples of continuous functions</u>

~~Ex Thm~~

Thm Suppose f & g are continuous at $x=a$. So are

1. $f+g$ 2. $f-g$ 3 cf 4. fg 5. $\frac{f}{g}$, $g(a)\neq 0$.

Proof Do one

So we get

Thm The following functions are continuous anywhere they are defined:

polynomials $(-\infty,\infty)$, Rational functions $\frac{p(x)}{q(x)}$ $q(x)\neq 0$, n^{th} root, trig, inverse trig, exp, log.

Problems

1. Show that $f(x)=\begin{cases} x^2+1 & x>3 \\ 1+3x & x\leq 3 \end{cases}$

is continuous on $(-\infty,\infty)$

2. Find c such that $f(x)=\begin{cases} \sin x & x>\pi/2 \\ x^2+c & x\leq \pi/2 \end{cases}$

is continuous on $(-\infty,\infty)$

3. Sketch a function continuous except at $x=-5$, and continuous from the left at $x=-5$.

4. How should we define $f(0)$ to make $f(x)=\frac{3\sin x}{x}$ continuous?

5. Where is $f(x) = \dfrac{\cos x + \sin x}{x^2 - 3x + 2}$ continuous?

6. Sketch a function w/ a jump disc. at $x = 1$ a removable at $x = 0$ and continuous elsewhere

7. Let $f(t)$ = temperature at time t in my office. Is $f(t)$ continuous?