Finding Horizontal Asymptotes

Review: $y=a$ is a H.A. if $\lim\limits_{x\to\infty} f(x)=a$ or $\lim\limits_{x\to-\infty} f(x)=a$.

* At most two hor. asymptotes.

Tails

1. $\lim\limits_{x\to\pm\infty} \frac{1}{x^r}=0$ when $r>0$ and x^r makes sense.

2. If $\frac{\infty}{\infty}$, divide all by highest power of x

3. If $\infty-\infty$, try to get as a fraction.

Ex

$$\lim_{x\to-\infty} \frac{2x^3+x+1}{6x^4+2} = \lim_{x\to-\infty} \frac{\frac{2}{x}+\frac{1}{x^3}+\frac{1}{x^4}}{6+\frac{2}{x^4}} = \frac{0}{6}=0$$

$$\lim_{x\to\infty} \frac{2x^4+x+1}{6x^3+2} = \lim_{x\to\infty} \frac{2+\frac{1}{x^3}+\frac{1}{x^4}}{\frac{6}{x}+\frac{2}{x^4}} = \infty$$

$$\lim_{x\to-\infty} \frac{\sqrt{5x^6-x^2}}{x^3+x-5} = \lim_{x\to-\infty} \frac{\frac{\sqrt{5x^6-x^2}}{x^3}}{1+\frac{x}{x^3}-\frac{5}{x^3}}$$

* When $x<0$ then $x^3=-\sqrt{x^6}$

$$= \lim_{x\to-\infty} \frac{\frac{\sqrt{5x^6-x^2}}{-\sqrt{x^6}}}{1+\frac{1}{x^2}-\frac{5}{x^3}} = \lim_{x\to-\infty} \frac{-\sqrt{5-\frac{1}{x^4}}}{1+\frac{1}{x^2}-\frac{5}{x^3}}$$

$$= \boxed{-\sqrt{5}}$$

Ex $\lim_{x \to \infty} \left(\sqrt{9x^2+x} - 3x\right) = \lim_{x \to \infty} \frac{\left(\sqrt{9x^2+x} - 3x\right)\left(\sqrt{9x^2+x} + 3x\right)}{\sqrt{9x^2+x} + 3x}$

$$= \lim_{x \to \infty} \frac{9x^2 + x - 9x^2}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x \to \infty} \frac{x}{\sqrt{9x^2+x} + 3x} = \lim_{x \to \infty} \frac{1}{\sqrt{9 + \frac{1}{x}} + 3}$$

$$= 1/6$$

PRECISE DEF OF LIMIT

Goal! Formal def of $\lim_{x \to a} f(x) = L$.

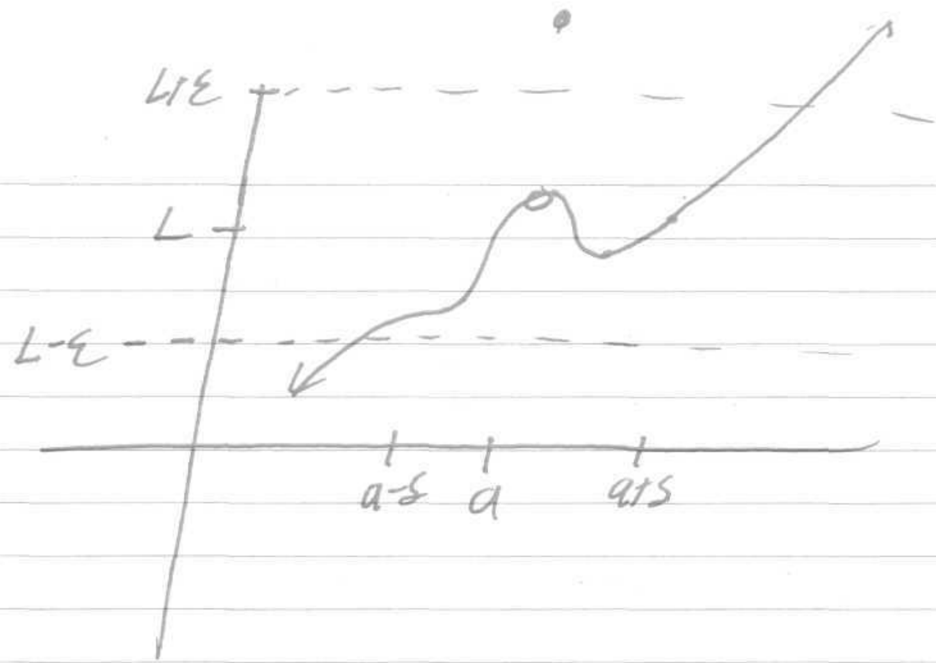
Limit Game

1. You choose $\varepsilon > 0$, however small you want.
2. I tell you how close to a you must get,

   i.e. I pick $\delta > 0$, so you look in $(a-\delta, a+\delta)$.

I must make it so for any c in $(a-\delta, a+\delta)$ except $c = a$, that $f(c)$ is ~~close~~ within ε of L.

Picture



* You go first, if you make ε smaller, I can make δ smaller.

Def $\lim_{x\to a} f(x) = L$ if for any $\varepsilon > 0$ there exists a $\delta > 0$ such that

if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$.

Rmk $\lim_{x\to a^+} f(x) = L$ if . . . . . – –

$0 < x-a < \delta$ . . . –

$\lim_{x\to a^-} f(x) = L$ if – – – –

$0 < a-x < \delta$ – – –

Example 1. Prove $\lim_{x\to a} x = a$

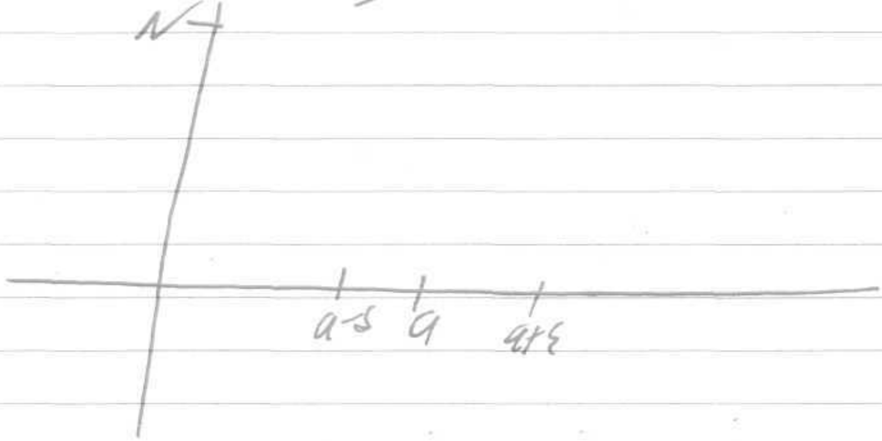
2. Prove $\lim_{x\to a} x^2 = a^2$

Infinite Limits

Def $\lim_{x \to a} f(x) = \infty$ if for any $N > 0$

there exists a δ such that

~~fexr~~ If $0 < |x-a| < \delta$ then $f(x) > N$.



Def $\lim_{x \to \infty} f(x) = L$ if for any $\varepsilon > 0$ there

exists $N > 0$ such that

if $x > N$ then $|f(x) - L| < \varepsilon$.