p203 # 30, 44, 63

p 213 # 2, 6, 12, 14

Review

Chain Rule: $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$

a.k.a. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Example

$$y = \cos\left(\sqrt{\sin(\tan(\pi x))}\right)$$

$$\frac{dy}{dx} = -\sin\left(\sqrt{\sin(\tan(\pi x))}\right) \cdot \frac{1}{2\sqrt{\sin(\tan(\pi x))}} \cdot \cos(\tan(\pi x)) \cdot \sec^2(\pi x) \cdot \pi$$

We used: $\frac{d}{dx}\cos x = -\sin x$ $\quad \frac{d}{dx}\sin x = \cos x$ $\quad \frac{d}{dx}\pi x = \pi$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ $\quad \frac{d}{dx}\tan x = \sec^2 x$

Ex $h(t) = (t^2+1)^3(2t+5)^{10}$

$$h'(t) = 3(t^2+1)^2 \cdot 2t\,(2t+5)^{10} + (t^2+1)^3 \cdot 10(2t+5)^9 \cdot 2$$

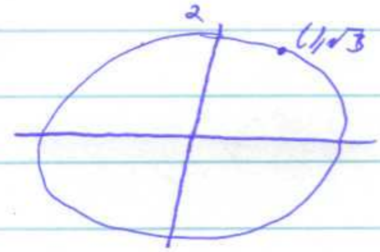
$$= 6t(t^2+1)^2(2t+5)^{10} + 20(t^2+1)^3(2t+5)^9$$

Ex $y = 3^{\sin(kx)}$ $\quad \frac{dy}{dx} = \left(3^{\sin(kx)}\ln 3\right) \cdot \cos(kx) \cdot k$

Problem: Suppose y and x are related by

$$y^2+x^2=4.$$

Find instantaneous rate of change of y with respect to x at point $x=1$ $y=\sqrt{3}$.



* Not given y as a function of x
Near $x=1$, y is a function of x.

$$y=\sqrt{4-x^2}$$

$$\frac{dy}{dx}=\frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)=\frac{-x}{\sqrt{4-x^2}}$$

$$\frac{dy}{dx}(x=1) \text{ is } \frac{-1}{\sqrt{3}}$$

Implicit Differentiation

1. Given relation involving (for example) x & y, want $\frac{dy}{dx}$.

2. Assume y is a function of x, $y=y(x)$, even if we can't solve for it.

3. Take derivative of both sides with respect to x.

4. Solve for $\frac{dy}{dx}$

Example $x^2+y^2=4$ Think: $x^2+y(x)^2=4$

$2x+2y\frac{dy}{dx}=0$ Applied $\frac{d}{dx}$ to both sides used chain rule on y^2.

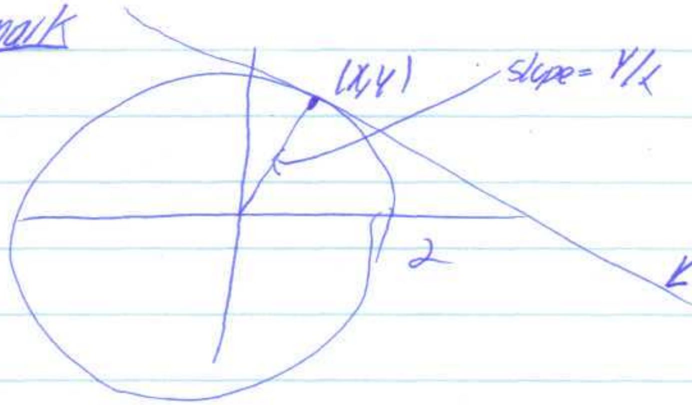
$\frac{dy}{dx}=\frac{-x}{y}$ solve for $\frac{dy}{dx}$

Remark In this example we can solve for y, we get

$y=\sqrt{4-x^2}$ $\frac{dy}{dx}=\frac{-x}{\sqrt{4-x^2}}=-\frac{x}{y}$

so answers agree (see p. 213 # 1-4)

Remark



This proves tangent line is perpendicular to radius!

Remark The 4 was not relevant!

Example

Find $\frac{dy}{dx}$

$$\cos(xy) + x + y = 5$$

$$-\sin(xy) \cdot \left(y + x\frac{dy}{dx}\right) + 1 + \frac{dy}{dx} = 0$$

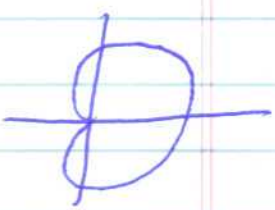
$$-y\sin(xy) - \frac{dy}{dx}x\sin(xy) + 1 + \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-1 + y\sin(xy)}{-x\sin(xy) + 1}}$$

ok that y's are in the answer, we can't solve for y anyhow

Example

Find eq of tangent line to the cardiod



$$x^2+y^2 = (2x^2+2y^2-x)^2 \text{ at } (0, \tfrac{1}{2})$$

$$2x + 2yy' = 2(2x^2+2y^2-x)(4x+4yy'-1) \qquad \left(y' = \frac{dy}{dx}\right)$$

easier to plug in before solving for y'

$$0 + y' = 2\left(0+\tfrac{1}{2}\right)(0+2y'-1)$$

$$y' = 2y' - 1$$

$$y' = 1$$

$$\boxed{y - \tfrac{1}{2} = x}$$

Ex

$xt + x\cos(xt) = 1$ Find $\frac{dx}{dt}$

$$\frac{dx}{dt}t + x + \frac{dx}{dt}\cos(xt) + x(-\sin(xt))\left(\frac{dx}{dt}t + x\right) = 0$$

$$\frac{dx}{dt}\left(t + \cos(xt) - xt\sin(xt)\right) + x - x^2\sin(xt) = 0$$

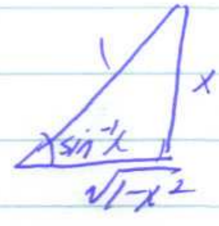
$$\frac{dx}{dt} = \frac{x^2\sin(xt) - x}{t + \cos(xt) - xt\sin(xt)}$$

Ex Let $y = \sin^{-1}x$ Find $\frac{dy}{dx}$

$y = \sin^{-1}x$

$\sin y = x$

$\cos y \frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\cos(\sin^{-1}x)}$



$= \frac{1}{\sqrt{1-x^2}}$

$$\boxed{\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}}$$