HW Read 2.3

p. 96 # 1, 18, 24, 37

8/28/07

Review $f(t) = 10t^2 + 10t \quad 0 \le t \le 5 \quad t = \text{hours}, \; f(t) \text{ in miles}$

Problem: How fast driving at time $t=3$.

Estimate: Average speed $t=3$ to $3.5 = \frac{DIST}{TIME} = \frac{f(3.5)-f(3)}{3.5-3} = 75\text{mph}$

Avg speed $t=3$ to $t=3.01 = \frac{f(3.01)-f(3)}{3.01-3} = \frac{120.701-120}{.01} = 70.1\text{mph}$

Avg speed $t=3$ to $t=3+h$ is

$$\frac{f(3+h)-f(3)}{3+h-3} = \frac{10(3+h)^2+10(3+h)-120}{h}$$

$$= (10(h^2+6h+9)+30+10h-120)/h$$

$$= 10h^2+70h/h$$

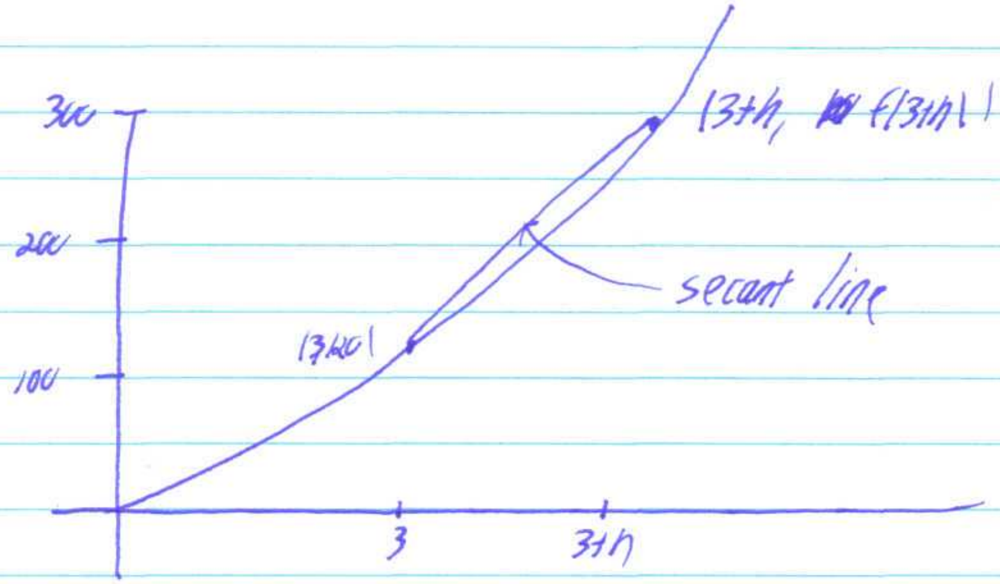
$$= 10h+70. \quad \text{Now } \lim_{h\to 0} 10h+70 = 70.$$

* instantaneous speed at $t=3$ is 70 mph.

Question: Why not replace 3 by a variable as well?

A: We will!

Let's interpret this calculation in terms of the graph



Key Observation: Avg rate of change $=$ Avg speed $= \frac{DIST}{TIME} = \frac{f(3+h)-f(3)}{3+h-3}$

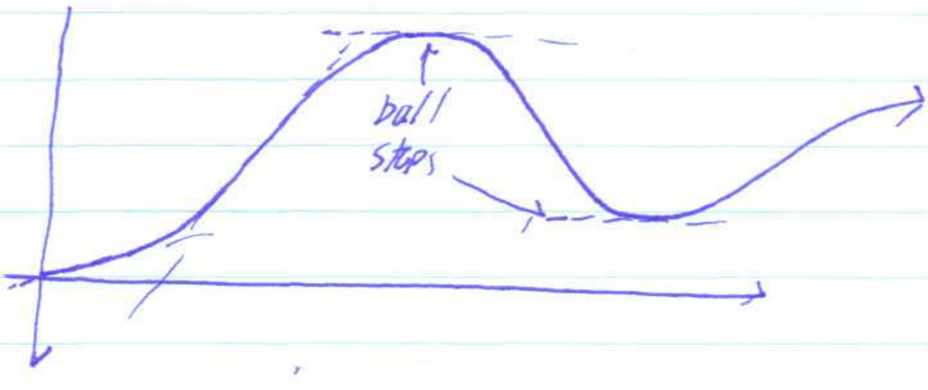
Slope of Secant $= \frac{\Delta Y}{\Delta X} = \frac{f(3+h)-f(3)}{3+h-3}$

* Determining average speed same as slope of secant line.

As $h \to 0$, secant line becomes tangent!

** Suppose $y=f(x)$. The <u>instantaneous rate of change</u> of y with respect to x at $x=a$ is slope of tangent at point $(a, f(a))$.

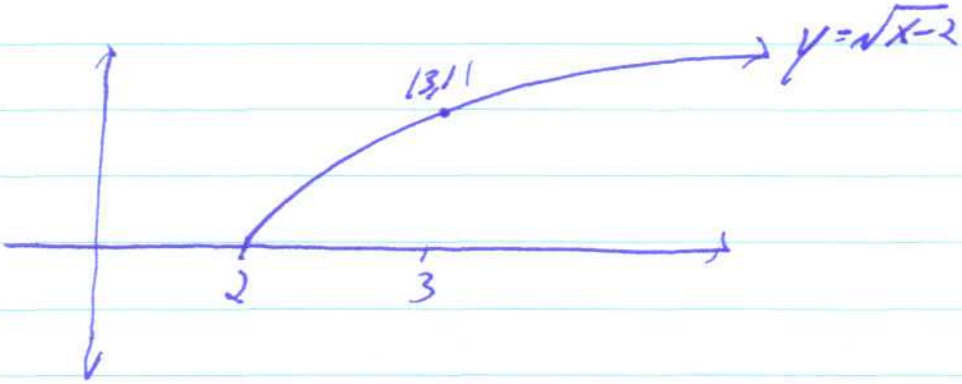
Example $h(t)$ = height of ball at time t.



Problem p.87 #4

$(3,1)$ lies on $y=\sqrt{x-2}$. 1. Find slope of secant for $x = 2.9, 2.99, 3.01, 3.1$

2. Guess slope of tangent
3. Find equation of tang line at $(3,1)$
4. Sketch



$x=2.9$ $\quad \frac{f(2.9)-f(3)}{2.9-3} = .513$ $\qquad x=2.99$ $\quad \frac{f(2.99)-f(3)}{2.99-3} - 5.012$

$x=3.1$ $\quad \frac{f(3.1)-f(3)}{3.1-3} = .488$ $\qquad x=3.01$ $\quad \frac{f(3.01)-f(3)}{3.01-f(3)}$ $\quad .498$

Guess $.5 = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$

3. $y-1 = .5(x-1)$

Limits

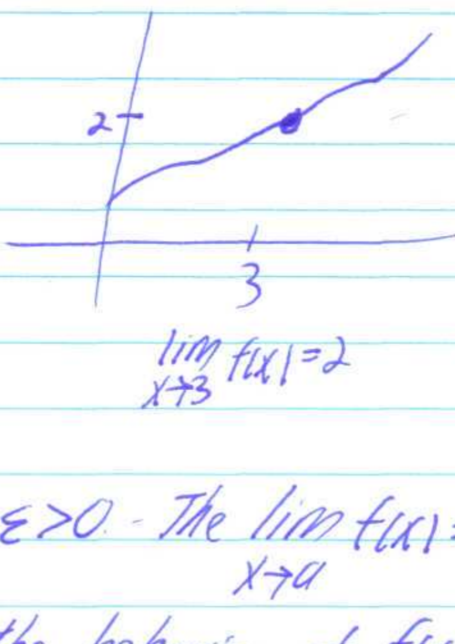
Plan: Find various ways to determine limits

Notation $\lim_{x\to a} f(x) = L$ = the limit as x approaches a of $f(x)$ is L "

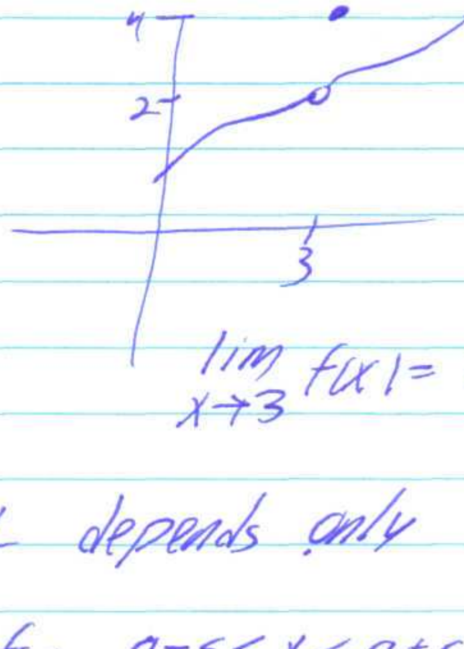
Formal Def to Follow...

Key Properties

1. $\lim_{x\to a} f(x) = L$ does not depend on $f(a)$, in fact $f(a)$ may be undefined!



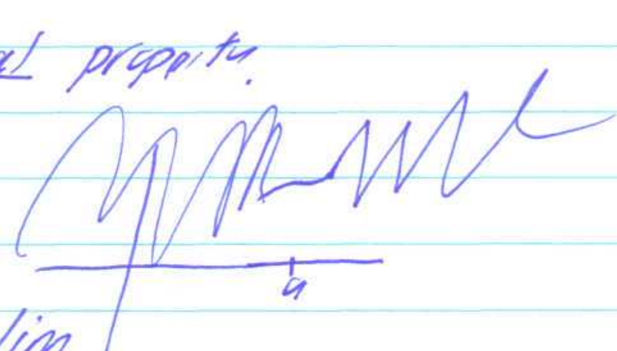
$\lim_{x\to 3} f(x) = 2$



$\lim_{x\to 3} f(x) = 2$

2. Let $\varepsilon > 0$. The $\lim_{x\to a} f(x) = L$ depends only on the behavior of $f(x)$ for $a-\varepsilon < x < a+\varepsilon$

It is a local property.



same $\lim_{x\to a}$