10/15/07

p. 214 # ~~[illegible]~~ 45, 53, 60
p. 220 # 2, 8, 10, 20, 30, 37, 42

Review

Implicit Diff Given equation relating x & y, can get $y' = \frac{dy}{dx}$ by ~~[illegible]~~ differentiating both sides with respect to x.

Example
Find tangent to ellipse $x^2+xy+y^2=3$ at (1,1)

$$2x + xy' + y + 2yy' = 3$$

$$y' = \frac{3-2x-y}{x+2y} \quad \text{at } (1,1) \quad y' = \frac{0}{3}$$

$$y-1=0 \quad \boxed{y=1}$$

Example
$x^3+y^3=1$ Find y''

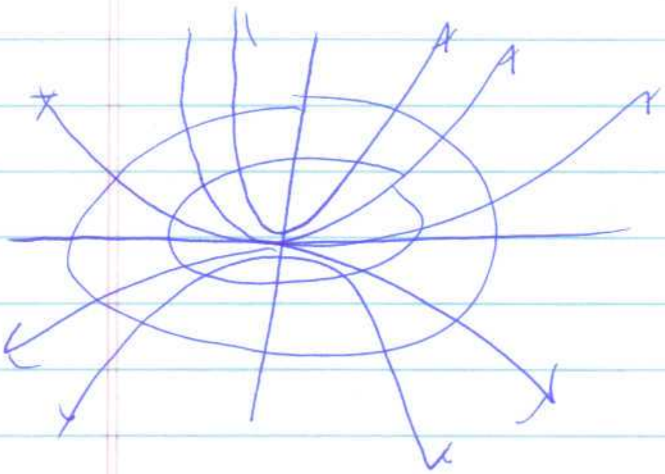
$$3x^2+3y^2y'=0 \qquad y' = \frac{-x^2}{y^2}$$

$$y'' = \frac{y^2(-2x) + x^2 2yy'}{y^4}$$

$$= \frac{-2xy^2 + 2x^2y\cdot\left(\frac{-x^2}{y^2}\right)}{y^4} = \frac{-2xy^2 - \frac{4x^2}{y}}{y^4}$$

$$= \frac{-2xy^3 - 4x^3}{y^5}$$

Ex Show parabolas $y=cx^2$ are all $\perp$ to ellipses $x^2+2y^2=k$



$y=cx^2$

$\boxed{y'=2cx}$

$x^2+2y^2=k$

$2x+4yy'=0$

$y'=\frac{-x}{2y}$

$=\frac{-x}{2cx^2}=\frac{-1}{2cx}$ ✓

DERIVATIVES OF LOGS

Theorem $\frac{d}{dx}(\log_a x)=\frac{1}{x\ln a}$

in particular $\frac{d}{dx}\ln x=\frac{1}{x}$

Proof

$y=\log_a x$

$a^y=x$

$a^y\ln a\cdot y'=1$

$y'=\frac{1}{a^y\ln a}=\boxed{\frac{1}{x\ln a}}$

Ex

$y = \ln(x^2) \qquad y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

$= 2\ln x$ so agrees!

Ex $f(x) = \ln(\sin x)$ Find domain $f(x)$

Find $f'(x)$

Ex $y' = \log_3((x^2 + \cos x))$

Logarithmic Diff

General idea: ln takes $\cdot \longrightarrow +$, $\div \longrightarrow -$

May be easier to take ln and use implic diff rather than diff.

Ex

$$y = \sqrt[4]{\frac{x^2+1}{x^2-1}} \quad \text{Find } y'.$$

$$\ln y = \frac{1}{4}\left(\ln|x^2+1| - \ln|x^2-1|\right)$$

$$\frac{1}{y}y' = \frac{1}{4}\left(\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right)$$

$$y' = \frac{1}{4}\sqrt[4]{\frac{x^2+1}{x^2-1}}\left(\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right)$$

Ex $y = (\tan x)^{1/x}$ $\quad \ln y = \frac{1}{x}\ln|\tan x|$

$$\frac{1}{y}y' = \frac{-1}{x^2}\ln(\tan x) + \frac{1}{x}\cdot\frac{\sec^2 x}{\tan x}$$

$$y' = (\tan x)^{1/x}\left(\quad '' \quad \right)$$

Ex $y = x^x$ $\quad \ln y = x\ln x$ $\quad \frac{1}{y}y' = \ln x + 1$

$$y' = x^x(\ln x + 1)$$