10/26 · Go over exam

CHPT 4: Applications of Differentiation

Major Application: Find value(s) that maximize or minimize a function $f(x)$, sometimes w/ some restriction on x.

Def Let $f(x)$ have Domain D. Then $f(x)$ has an absolute maximum (aka ~~local~~ global maximum) at c if

$$f(c) \geq f(x) \quad \forall x \in D$$

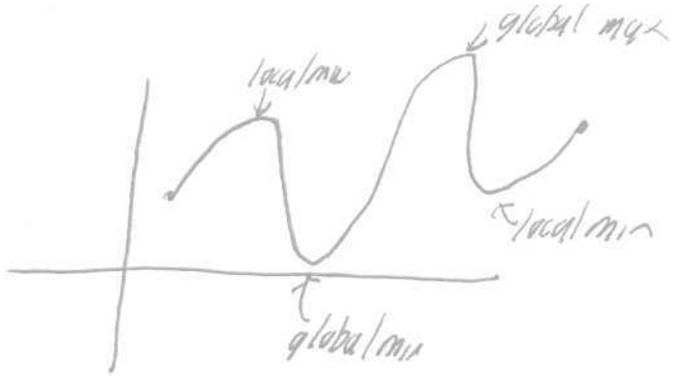
The # $f(c)$ is the maximum value.

Extreme value = either one

Similarly f has an absolute minimum at c if

$$f(c) \leq f(x) \quad \forall x \in D \qquad f(c) \text{ is minimum value}$$

Ex



Def $f(x)$ has a local maximum at c if $\exists\, \varepsilon > 0$ so that $f(c) \geq f(x)$ for all $x \in (c-\varepsilon, c+\varepsilon)$

Similarly local minimum

Ex

Ex $f(x) = x^2$ has a global (and local) min at $x = 0$.
On $(-\infty, \infty)$ it has no global maximum.

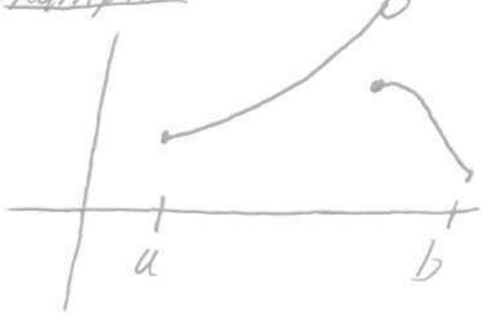
$f(x) = x^2$ on $[-2, 5]$ has a global maximum value of 25 at $x = 5$.

Two Key Theorems:

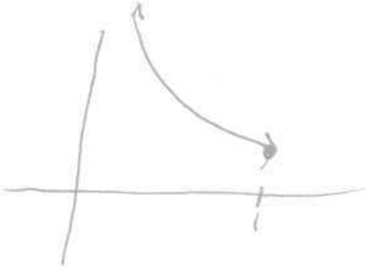
<u>Extreme Value Thm</u>

Let $f(x)$ be <u>continuous on a closed interval</u> $[a,b]$. Then $f(x)$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some $c, d \in [a,b]$

<u>Examples</u>



No maximum value!



$f(x) = \frac{1}{x}$ on $(0, 1]$

No maximum value,
Not a closed interval!

Fermat's Theorem

Suppose $f(x)$ has a local max or min at c.
If $f'(c)$ exists then $f'(c)=0$.

"Proof" Suppose local max at c.

For $h>0$ and small, $\dfrac{f(c+h)-f(c)}{h} < 0$
For $h<0$ and small, $\dfrac{f(c+h)-f(c)}{h} > 0$

Thus $\lim\limits_{h\to 0} \dfrac{f(c+h)-f(c)}{h}$ must $=0$ if it exists
$\qquad \| $
$\quad f'(c)$

Ex $f(x)=x^2$

$f'(x)=2x$ Thus local max/min can only occur at $x=0$

Ex $f(x)=x^3$ $f'(x)=3x^2$ but no local max/min at $x=0$