P.285 # 2, 8, 13, 14, 16, 18

Review Suppose f(x) is continuous on [a,b]. To find global max/min:

1. Find critical #'s (where $f'=0$ or $f'$ DNE)

2. Plug in critical #'s and endpoints ($x=a,b$).

3. Compare, largest value is global max
smallest is global min

Ex $f(x) = x^{-2}\ln x = \frac{\ln x}{x^2}$ on $[1,4]$

$$f'(x) = \frac{x^2(\frac{1}{x}) - 2x\ln x}{x^4} = \frac{x(1-2\ln x)}{x^4} = \frac{1-2\ln x}{x^3}$$

$1-2\ln x = 0 \quad \frac{1}{2} = \ln x \quad x = e^{1/2}$

| $x$ | $f(x)$ | |
|---|---|---|
| $1$ | $0$ | |
| $\sqrt{e}$ | $1/2e = .1839..$ | Global max at $x=\sqrt{e}$ |
| $4$ | $\ln 4/16 = .086..$ | Global min at $x=1$ |

Rolles Thm Let $f(x)$ be continuous on $[a,b]$, differentiable on $(a,b)$ and such that $f(a)=f(b)$.

Then there is a number $c\in(a,b)$ with $f'(c)=0$.

Proof Last time

Application Prove $x^3+x-1=0$ has exactly one real root.

Proof Let $f(x)=x^3+x-1$. Since $f(0)=-1$, $f(1)=1$

Then $f$ has a root in $(0,1)$ by the intermediate value thm

Suppose $f$ has 2 roots, so $f(a)=f(b)=0$. Then by Rolle's, $f'(c)=0$ for some $c$. But

$f'(x)=3x^2+1$ is always $>0$ $\neq$.

Mean Value Thm

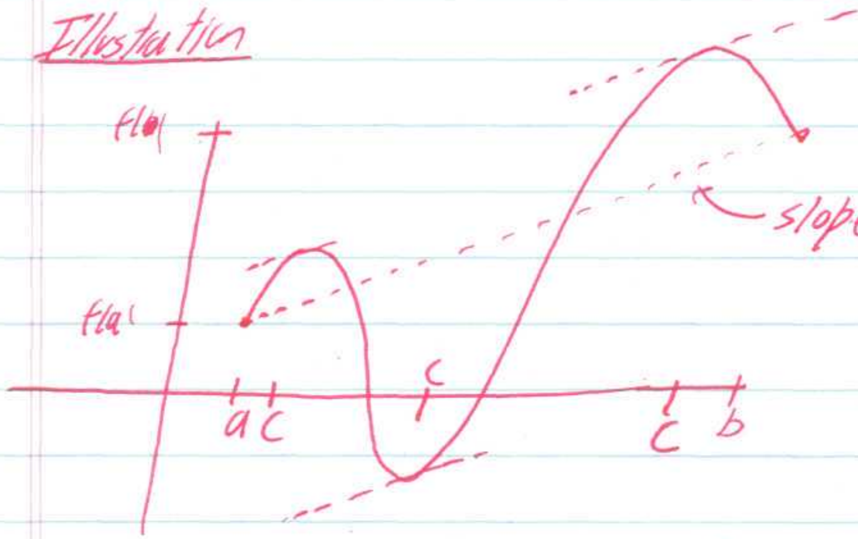
Suppose $f$ is continuous on $[a,b]$ and differentiable on $(a,b)$. Then there exists a number $c$ in $(a,b)$ with

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

<u>Remark</u> $\frac{f(b)-f(a)}{b-a}$ is <u>average</u> rate of change of $f(x)$ for $a \le x \le b$

$f'(c)$ is <u>instantaneous rate of change</u> at $x=c$.

<u>Illustration</u>




Three possible c's!
MVT only guarantees 1.

<u>Example</u> $f(x)=3x^2+2x+5$ on $[-1,1]$.
Check $f(x)$ satisfies hypotheses of MVT. Find all c that work.

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{10-6}{2} = 2$$

$f'(x) = 6x+2$

$2 = 6x+2$

$x=0$ so $c=0$ has $f'(c)=2$.

COROLLARY

If $f'(x)=0$ for all $x\in(a,b)$ Then $f$ is constant on $(a,b)$.

Proof Suppose not, so $f(x_2)\neq f(x_1)$ some $x_1\neq x_2\in(a,b)$.
Then MVT guarantees $c$ w/
$$f'(c)=\frac{f(x_2)-f(x_1)}{x_2-x_1}\neq 0 \quad \divideontimes$$

COR If $f'(x)=g'(x)$ for all $x\in(a,b)$ Then
$$f(x)=g(x)+C.$$

Example
$$f(x)=\frac{|x|}{x} \quad \text{on } [-1,1]$$
What goes wrong?

Ex Prove that $\sqrt{1+x}<1+\frac{1}{2}x$ if $x>0$

Ex Prove a poly of degree $n$ has at most $n$ roots