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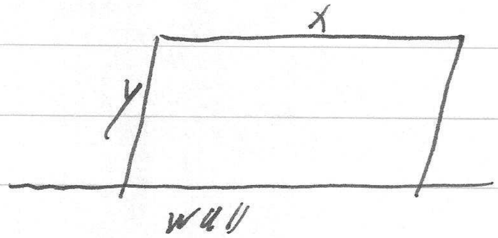
OPTIMIZATION

Problem: These are just max/min word problems
1. Get equation for what is being optimized
   - sketch
   - eliminate variables?
   - constraints (Domain)
2. Use calculus

Example I have 100 feet of fence to make rectangular pen against a wall. What is largest area I can enclose?



$A = xy$ but
$2y + x = 100$
so $x = 100 - 2y$

$A(y) = (100-2y)y = 100y - 2y^2 \qquad 0 \le y \le 50$

$A'(y) = 100 - 4y$

$100 - 4y = 0$
$y = 25$

y	$A(y)$
0	0
25	1250
50	0

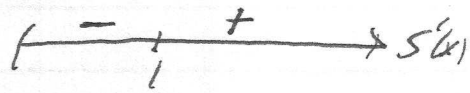
max area is 1250 sq feet.

Ex Find a positive # s.t. sum of number and its reciprocal is as small as possible.

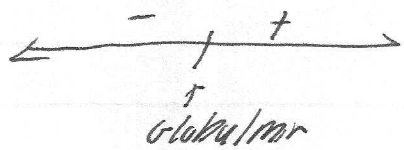
Minimize $S(x) = x + \frac{1}{x}$ w/ $x > 0$

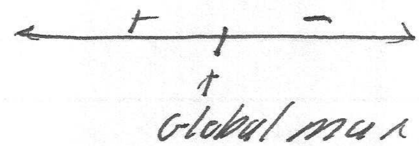
$S'(x) = 1 - \frac{1}{x^2}$

$0 = 1 - \frac{1}{x^2}$ $\quad x = \pm 1$

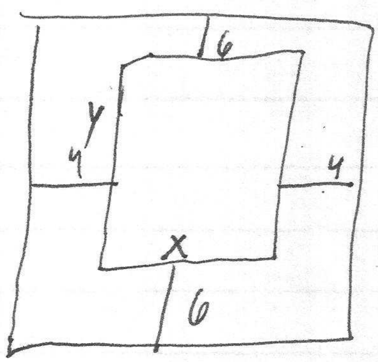


Thus S(x) has a <u>global</u> min at $x = 1$.





Ex The top and bottom margins of a poster are each 6 cm, the side margins are 4 cm. If the area of printed material is fixed at 384 cm², Find dimensions of poster w/ smallest area.



Given $xy = 384$

Minimize $(x+8)(y+12)$

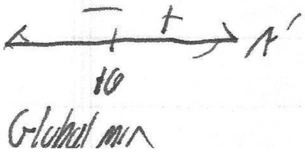
Minimize $A(x) = (x+8)\left(\frac{384}{x} + 12\right)$

$= 384 + 12x + \frac{3072}{x} + 96$

$A(x) = 480 + 12x + 3072x^{-1}$

$A'(x) = 12 - \frac{3072}{x^2}$ $\quad$ set $A'(x) = 0$, $x = 16$

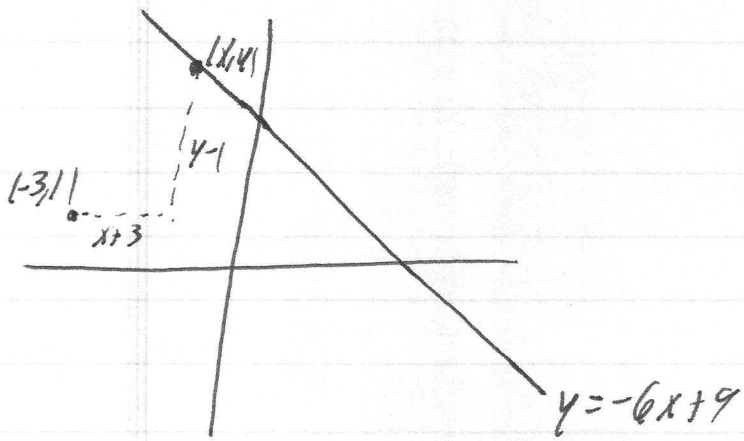
so $y = 24$



$\boxed{\text{Answer } 24 \times 36}$

Ex

Find point on the line $6x + y = 9$ closest to $(-3, 1)$



Minimize $D = \sqrt{(x+3)^2 + (y-1)^2}$ with $y = -6x+9$

$$D(x) = \sqrt{(x+3)^2 + (-6x+8)^2}$$

Trick: Minimize $D^2 = (x+3)^2 + (-6x+8)^2$

$$(D^2)' = 2(x+3) - 12(-6x+8)$$

$$= \cancel{-80x - 80}$$

$\cancel{0 = -70x - 80 \quad x = -9/7}$

$$= 74x - 90$$

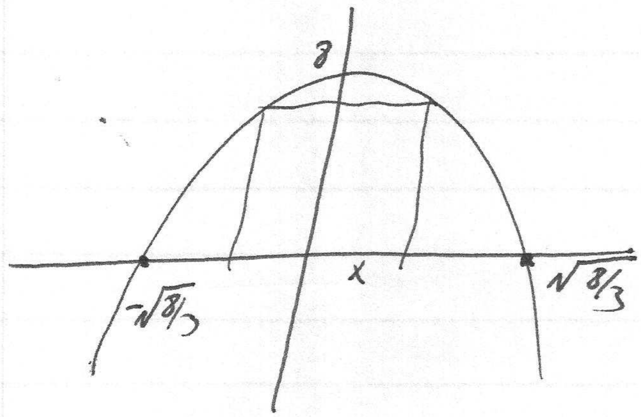
$$x = \frac{90}{74}$$

$$x = \frac{45}{37} \quad \text{Global min}$$

$$y = -6\left(\frac{45}{37}\right) + 9$$

Answer $\left(\frac{45}{37}, \frac{-6 \cdot 45}{37} + 9\right)$

4. Find largest rectangle w/ base on x-axis under $y = 8-3x^2$



Maximize $A(x) = 2x(8-3x^2) \qquad 0 \le x \le \sqrt{8/3}$

$$= 16x - 6x^3$$

$$A'(x) = 16 - 18x^2$$

$$x = \pm\sqrt{16/18} = \pm\frac{4}{3\sqrt{2}}$$

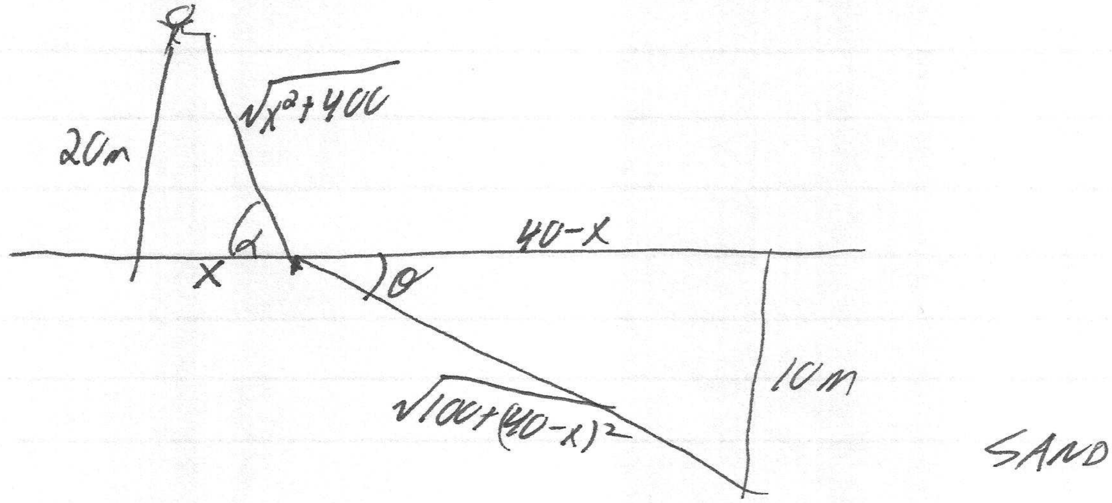
x	$A(x)$
0	0
$\frac{4}{3\sqrt{2}}$	$+$
$\sqrt{8/3}$	0

$$2\cdot\frac{4}{3\sqrt{2}} \times 8 - 3\cdot\frac{16}{18}$$

Dimensions $\boxed{\frac{8}{3\sqrt{2}} \times \frac{96}{18}}$

<u>Problem</u>

A lifeguard is 10 m from water. A swimmer is 20m from shore and 40m sideways from lifeguard. She runs 5 m/sec, swims 2 m/sec. Minimize time to get to swimmer



$$\text{Total Time} = T(x) = \frac{\sqrt{100+(40-x)^2}}{5} + \frac{\sqrt{x^2+400}}{2}$$

$$T'(x) = \frac{1}{5}\cdot\frac{-2(40-x)}{2\sqrt{100+(40-x)^2}} + \frac{1}{2}\cdot\frac{2x}{2\sqrt{x^2+400}}$$

$$= \frac{-1}{5}\cos\theta + \frac{1}{2}\cos\alpha$$

So if $T'(x)=0$ then

$$\frac{\cos\alpha}{\cos\theta} = \frac{2}{5}$$

SNELL'S LAW