p. 97 #4, 6, 8, 12, 14

Lecture 3

Review

We are learning to calculate $\lim_{x\to a} f(x)$, the limit as x approaches a of f(x).

Properties/Informal Def

1. f(a) is irrelevant to $\lim_{x\to a} f(x)$, only function near but not at $x=a$ matters

2. Limit is determined completely by f(x) for $a-\varepsilon < x < a+\varepsilon$ for any $\varepsilon > 0$. Limit is a "local" property

3. Informally $\lim_{x\to a} f(x) = L$ if we can make f(x) arbitrarily close to L by taking x sufficiently close to a on either side.

$\lim_{x\to -1} f(x)$

Example Guess value of $f(x) = \frac{x^2-1}{x^2+3x+2}$

Notice f(-1) undefined

x	$f(x)$
-1.1	~~-2~~ -2.333
-1.01	-2.0303
-.99	-1.97
-.9	-1.7

Guess $\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2} = -2$

Notice $\frac{x^2-1}{x^2+3x+2} = \frac{(x+1)(x-1)}{(x+1)(x+2)}$

$= \frac{x-1}{x+2}$ except at $x=-1$

Thus $\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2} = \lim_{x\to -1} \frac{x-1}{x+2} = -2$

<u>Example</u>

Guess $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

$t = .0001 \quad .16653$

Answer is $1/6 = .1\overline{6}$

Book claims calculator will let you down.
Depends on your calculator!
Graphing calc/Maple may let you down too!

<u>Example</u> $\lim_{x\to 0} \frac{\sin x}{x}$

1. $\frac{0}{0}$ at $x=0$, so fcn DNE
2. Be sure to use radians!

$\lim_{x\to 0} \frac{\sin x}{x} = 1$ (Proof later!)

A warning Example

Let $f(x) = \sin(\pi/x)$ defined for $x \neq 0$.

x	$f(x)$
$1/2$	0
$1/4$	0
$1/6$	0
$1/8$	0
$1/10$	0

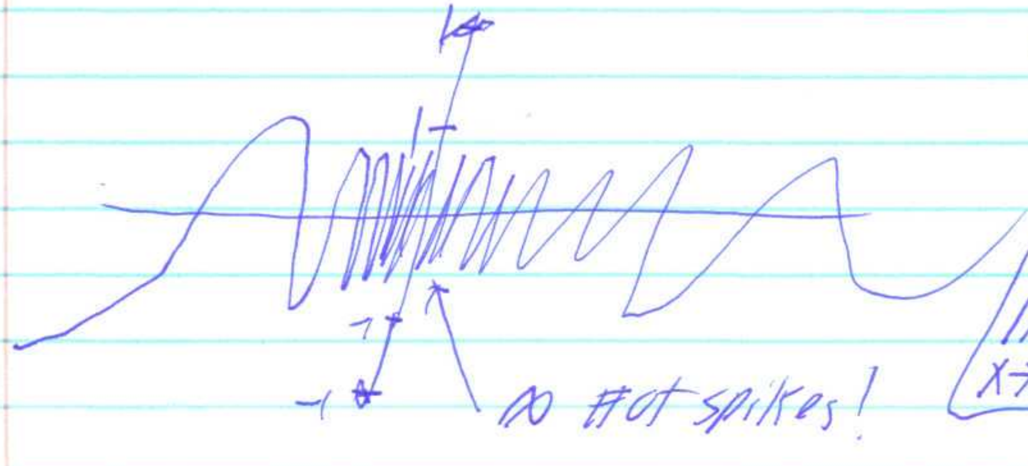
looks like $\lim_{x \to 0} \sin(\pi/x) = 0$!

x	$f(x)$
2	1
$2/5$	1
$2/9$ ~~2/10~~	1
$2/13$	1
$2/17$	1

looks like $\lim_{x \to 0} \sin(\pi/x) = 1$!!

As $x \to 0$, $\pi/x \to \infty$, so $\sin(\pi/x)$ goes up and down infinitely often

$y = \sin(\pi/x)$

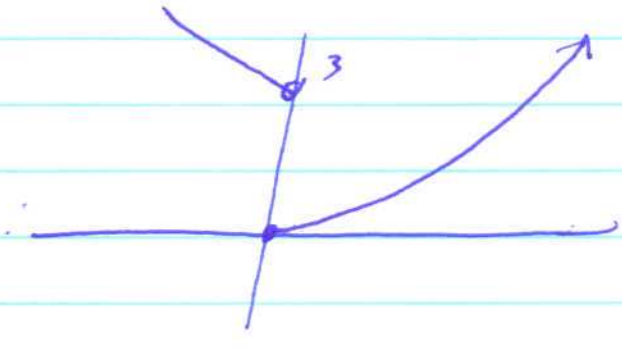


∞ # of spikes!

$\boxed{\lim_{x \to 0} \sin(\pi/x) \text{ DNE}}$

Example

$$f(x) = \begin{cases} 3-x & x<0 \\ x^2 & x \geq 0 \end{cases}$$



Notice $\lim_{x\to 0} f(x)$ DNE. We say

$$\lim_{x\to 0^+} f(x) = 0 \qquad \lim_{x\to 0^-} f(x) = 3$$

Again $f(0)$ is irrelevant!

Fact $\lim_{x\to a} f(x) = L$ if & only if $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$

Ex