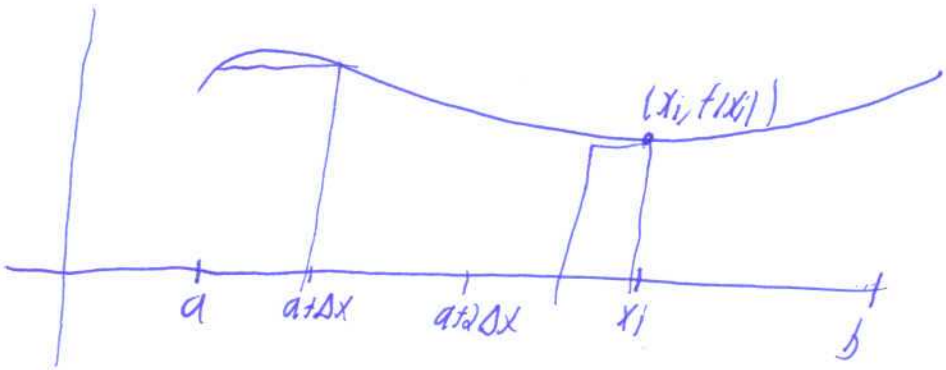
p. 374 #2, 6, 18, 23, 33, 34, 35, 36

Review

Given a continuous function $f(x) \geq 0$ on $[a,b]$



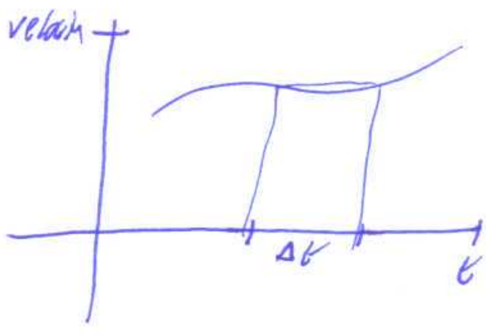

$$\Delta x = \frac{b-a}{n} \qquad x_i = a + i\Delta x$$

Approximation to area $R_n = \sum_{i=1}^{n} f(x_i)\Delta x$

$$\text{Area} = \lim_{n\to\infty} R_n$$

Example

Suppose $f(t)$ = velocity, $t$ = time..




Then $f(t_i)\Delta t$ = velo · time
= distance

* Sum of areas approximates distance

* Limit will be the distance.

Example Given table / graph, estimate Distance

# Definite Integrals

Def Let $f$ ~~have~~ be defined on $[a,b]$.
Divide $[a,b]$ into $n$ intervals of width $\Delta x = \frac{b-a}{n}$
Choose $x_i^* \in [x_{i-1}, x_i]$. The definite integral of $f$ from $a$ to $b$ is

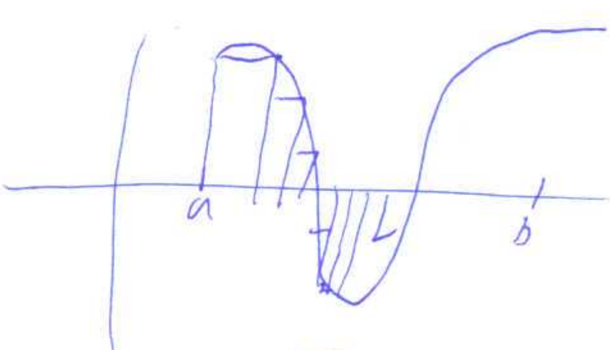
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

if it exists. If so, say $f$ is integrable on $[a,b]$.

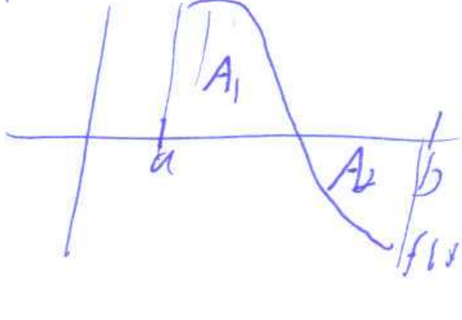
Words

- $\int$ integral sign, think "S" = sum
- $a, b$ are lower and upper limits of integration
- $\sum f(x_i^*)\Delta x$ is a Riemann sum

Interpretation




Area beneath the $x$-axis will count with a negative sign




$$\text{Area } A_1 - \text{Area } A_2 = \int_a^b f(x)\,dx$$

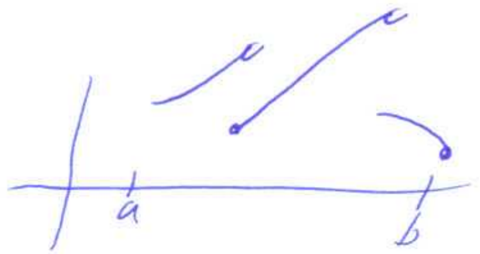
<u>Example</u> Find $\int_2^6 2x-6\,dx$

<u>Ex</u> Find $\int_0^4 \sqrt{16-x^2}\,dx$

<u>Properties</u>

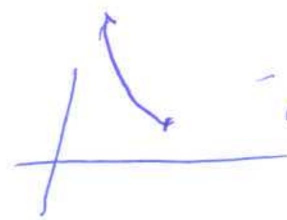
1. Any function with finitely many jump discontinuities is integrable.

<u>Ex</u>




yes

Ex.



$f(x) = 1/x$ on $[0,1]$

?? not jump, thm doesnt apply

2. If $f$ is integrable, you can always calculate integral using equally space intervals and right or left endpoints

Ex $\int_0^2 x^3\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(0+i\cdot\frac{2}{n})\cdot\frac{2}{n}$

$= \lim_{n\to\infty} \frac{2}{n} \cdot \sum_{i=1}^{n} \frac{8i^3}{n^3}$

$= \lim_{n\to\infty} \frac{16}{n^4} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{16}{n^4}\left(\frac{n(n+1)}{2}\right)^2$

$= \frac{16(n^4+2n^3+n^2)}{4n^4}$

$= 4$

Ex Set up but do not evaluate a limit for

$\int_2^7 e^x\,dx$