12/3 p. 406 #6, 12, 14, 21, 25, 32, 51, 54, 65

Situation:

We want to take antiderivatives but we can only do ones on our short list:

Ex $\frac{d}{dx}\sec x = \sec x\tan x$ so $\int \sec x\tan x\,dx = \sec x + C$

What about $\int x\cos(x^2)\,dx$? $x\cos(x^2)$ is not on our list of derivatives!

Ex $\int x\cos(x^2)\,dx$

looks like $x\cos(x^2)$ came from $\sin(x^2)$

Tool: Let $u = x^2$

$du = 2x\,dx$ (differential since $\frac{du}{dx} = 2x$)

$$\int \cos(x^2)\cdot x\,dx = \int \cos(u)\cdot \tfrac{1}{2}\,du$$
$$= \tfrac{1}{2}\int \cos u\,du = \tfrac{1}{2}\sin u + C$$
$$= \tfrac{1}{2}\sin(x^2) + C$$

Easy to check: Take derivative of $\frac{1}{2}\sin(x^2)+C$ to get $x\cos(x^2)$.

Recall Suppose $u = g(x)$. Then derivative of $f(u)$ is $f'(g(x))\cdot g'(x)$ so

Substitution Rule Suppose $u = g(x)$ is diff'ble. Then

$$\int f(g(x))\cdot g'(x)\,dx = \int f(u)\,du$$

Ex $\int x^2(3x^3+7)^{10}\,dx$

$u = 3x^3+7 \quad du = 6x^2\,dx$

$\int \frac{1}{6}u^{10}\,du = \frac{1}{66}u^{11} + C = \frac{1}{66}(3x^3+7)^{11} + C$

u-substitution Procedure

1. Choose $u = g(x)$ (need to be strategic!)
2. Calculate $du = g'(x)\,dx$
3. Replace all "x's" w/ u
4. Integrate
5. Put back as x's

Ex $\int e^{2x}\,dx \qquad u = 2x \quad du = 2\,dx$

$\int \frac{1}{2}e^{u}\,du = \frac{1}{2}e^{u} + C = \frac{1}{2}e^{2x} + C$

Recall $\int \frac{1}{x}\,dx = \ln|x| + C$ (covers $x<0$ or $x>0$)

Ex $\int \frac{5x}{x^2+3}\,dx \qquad u = x^2+3 \quad du = 2x\,dx$

$5x\,dx = \frac{5}{2}\,du$

$\int \frac{5}{2}\cdot\frac{1}{u}\,du = \frac{5}{2}\ln|u| + C$

$= \frac{5}{2}\ln|x^2+3| + C$

Ex $\int \sqrt{2x+1}\,dx$

$u=2x+1 \quad du=2\,dx$

$\int \frac{1}{2} u^{1/2}\,du = \frac{1}{3} u^{3/2} + C$

$= 1/3\,(2x+1)^{3/2} + C$

Ex $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx \qquad u=\cos x$

$du = -\sin x\,dx$

$= \int -\frac{1}{u}\,du = -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln\left|\frac{1}{\cos x}\right| + C$

$\int \tan x\,dx = \ln|\sec x| + C$

Definite Integrals

$\int_1^3 x e^{x^2}\,dx \qquad u=x^2 \quad du=2x\,dx$

$\int \frac{1}{2} e^u\,du = \frac{1}{2} e^u \Big|_{u=1}^{u=9} = \frac{1}{2}(e^9 - e)$

$x=1 \rightarrow u=1$

$x=3 \rightarrow u=9$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$$

$$\int \frac{\cos x}{1+\sin^2 x}\,dx \qquad u = \sin x \quad du = \cos x\,dx$$

$$\int \frac{du}{1+u^2} = \boxed{\tan^{-1}(\sin x) + C}$$

<u>EX</u> $$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} \qquad u = \ln x \quad du = \frac{1}{x}\,dx$$

$$= \int u^{-1/2}\,du = 2u^{1/2}$$

$$= 2\sqrt{\ln x}\,\Big|_e^{e^4}$$

$$= 2\sqrt{4} - 2\sqrt{1}$$

$$= 2$$