p. 117 # 4, 15, 18
also 4ab

Review

Def $\lim_{x\to a} f(x) = L$ if for all $\varepsilon > 0$ there exists a $\delta > 0$ such that $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

Example

Prove that $\lim_{x\to 6} (3x-5) = 13$

Work Let $\varepsilon > 0$. We need $\delta > 0$ so that

$$\text{if } 0 < |x-6| < \delta \text{ then } |3x-5-13| < \varepsilon$$

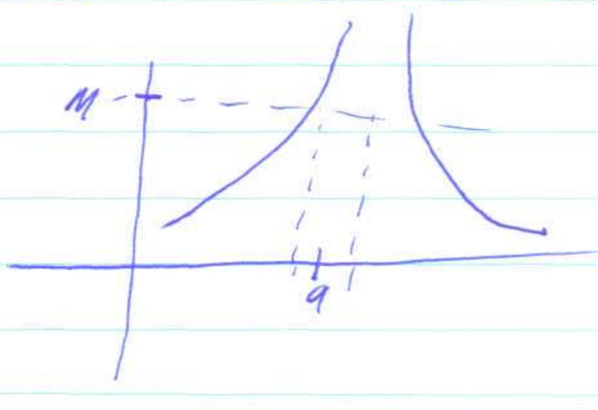
$$|3x-5-13| = |3x-18| = 3|x-6| \qquad \text{choose } \delta = \varepsilon/3$$

Proof Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$.
Suppose $0 < |x-6| < \varepsilon/3$. Then
$|f(x) - 13| = |3x-18| = 3|x-6| < 3 \cdot \varepsilon/3 = \varepsilon$. //

Two similar definitions

Infinite Limits:

Say $\lim_{x\to a} f(x) = \infty$ if for any $M > 0$ there exists $\delta > 0$ such that if $0 < x-a < \delta$ then $f(x) > M$.



Similar for

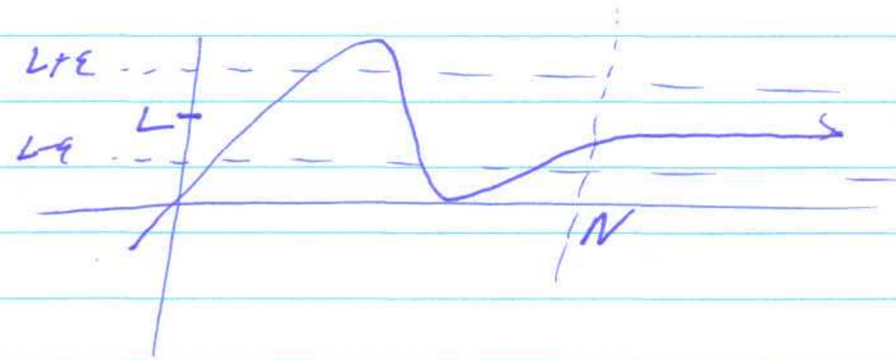
$\lim_{x\to a^+} f(x) = \infty$

$\lim_{x\to a} f(x) = \infty$

Limits at ∞

Def Say $\lim_{x\to\infty} f(x) = L$ if for any $\varepsilon > 0$ there exists $N > 0$ such that

if $x > N$ then $|f(x) - L| < \varepsilon$.



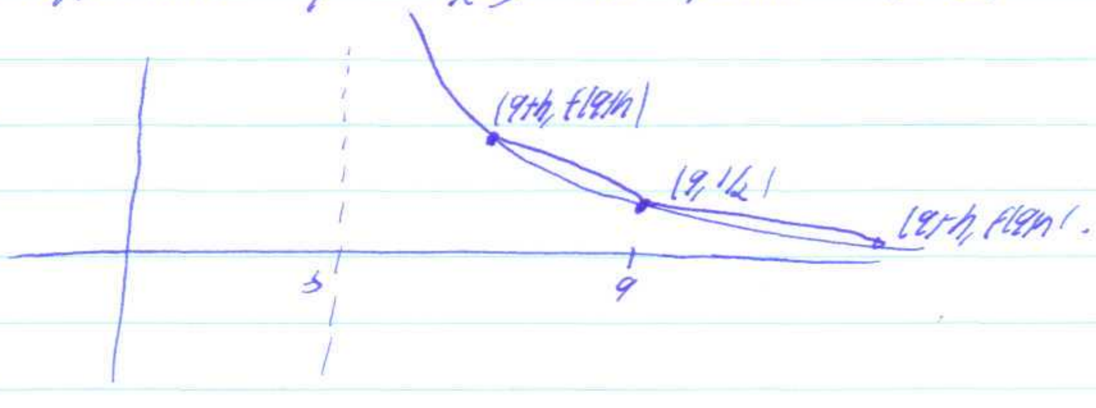
Review

1 Started w/ problem of calculating instantaneus rate of change. This led us to taking limits of avearge rate of change.

2 We have a formal definition of limits now.

3 Limits were used to give a precise definition of the notion of continuity

Problem Find the equation of the tangent line to the hyperbola $y = \frac{2}{x-5}$ at point $(9, 1/2)$.



Slope of secant: $\frac{f(9+h)-f(9)}{9+h-9} = \frac{\frac{2}{9+h-5} - 1/2}{h} = \frac{\frac{2}{4+h} - 1/2}{h}$

Slope of tangent $= \lim_{h\to 0} \frac{f(9+h)-f(9)}{h} = \lim_{h\to 0} \frac{\frac{2}{4+h} - 1/2}{h}$

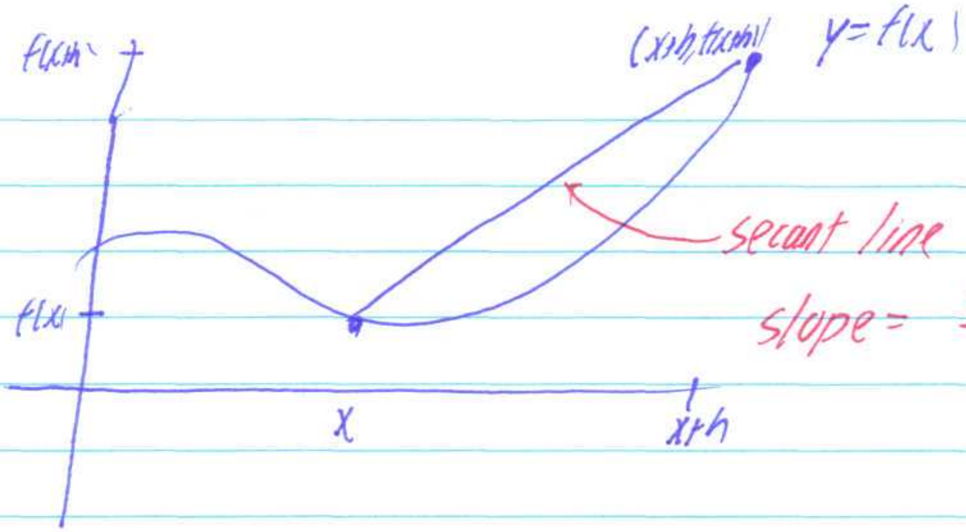
$= \lim_{h\to 0} \frac{\frac{4}{8+2h} - \frac{4+h}{8+2h}}{h}$

$= \lim_{h\to 0} \frac{\frac{-h}{8+2h}}{h} = \lim_{h\to 0} \frac{-1}{8+2h}$

$= -1/8$

Thus equation is $\boxed{y - 1/2 = -1/8\,(x-9)}$

Why focus on $x=9$? How about find the slope at $(x, f(x))$.



slope = $\frac{f(x+h) - f(x)}{x}$

Definition Let $f(x)$ be a function. The derivative of f is a new function, $f'(x)$, defined as

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

* The slope of the tangent line to the graph $y = f(x)$ at a point $(a, f(a))$ is given by $f'(a)$.

Example

Suppose $f(x)=x^2$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h)}{h}=2x.$$

$f(x)=x^2 \quad f'(x)=2x.$

Problem Find tangent line to $y=x^2$ at $(-3,9)$.

Slope $= f'(-3) = -6$

$$\boxed{y-9=-6(x+3)}$$