Name: SOLUTIONS

Math 141- Midterm Exam #1 - September 24, 2007

1. **(15 points)** True or false:

F a. A function which is continuous at $x = a$ must also be differentiable at $x = a$.

T b. It is possible for the graph of a function to have 3 vertical asymptotes.

F c. The intermediate value theorem applies to $f(x) = 1/x$ on the interval $[-2, 1]$.

F d. If $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$ then $\lim_{x\to 0}[f(x) - g(x)] = 0$

T e. If $p(x)$ is a polynomial then $\lim_{x\to 5} p(x) = p(5)$.

2. **(20 points)**

a. Give the formal definition for $\lim_{x\to a} f(x) = L$.

For any $\varepsilon > 0$ there is a $\delta > 0$ such that

if $0 < |x-a| < \delta$ then $|f(x)-L| < \varepsilon$.

b. Use the definition to prove that

$$\lim_{x\to 4}(3x - 7) = 5.$$

Let $\varepsilon > 0$. Choose $\delta = \varepsilon/3$.

Suppose $0 < |x-4| < \delta$. Then

$$|f(x)-L| = |3x-7-5| = |3x-12| = 3|x-4| < 3\delta = \varepsilon.$$

Thus $|f(x)-L| < \varepsilon$

as required. //

3. (**20 points**) Evaluate the following limits. If the limit does not exist then write DNE.

a. $\lim_{x \to -3} \frac{x^2-9}{x^2+2x-3}$

$$= \lim_{x \to -3} \frac{(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x \to -3} \frac{-6}{-4}$$

$$= 3/2$$

b. $\lim_{x \to 0} \frac{|x|}{x}$

DNE

c. $\lim_{x \to -\infty} \frac{\sqrt{x^2-9}}{2x-6}$.

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{x^2-9}}{x}}{2-6/x} = \lim_{x \to -\infty} \frac{-\sqrt{1-9/x^2}}{2-6/x}$$

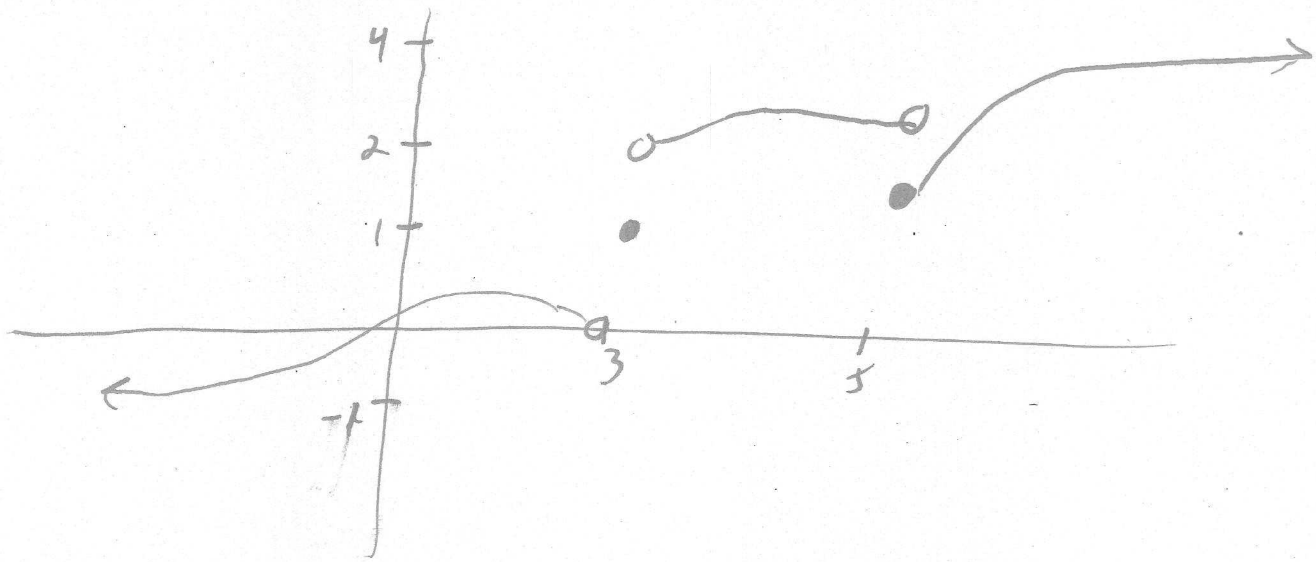
$$= -1/2$$

d. $\lim_{x \to \infty} \frac{2x^2-5x+11}{x^2-2}$.

$$= 2$$

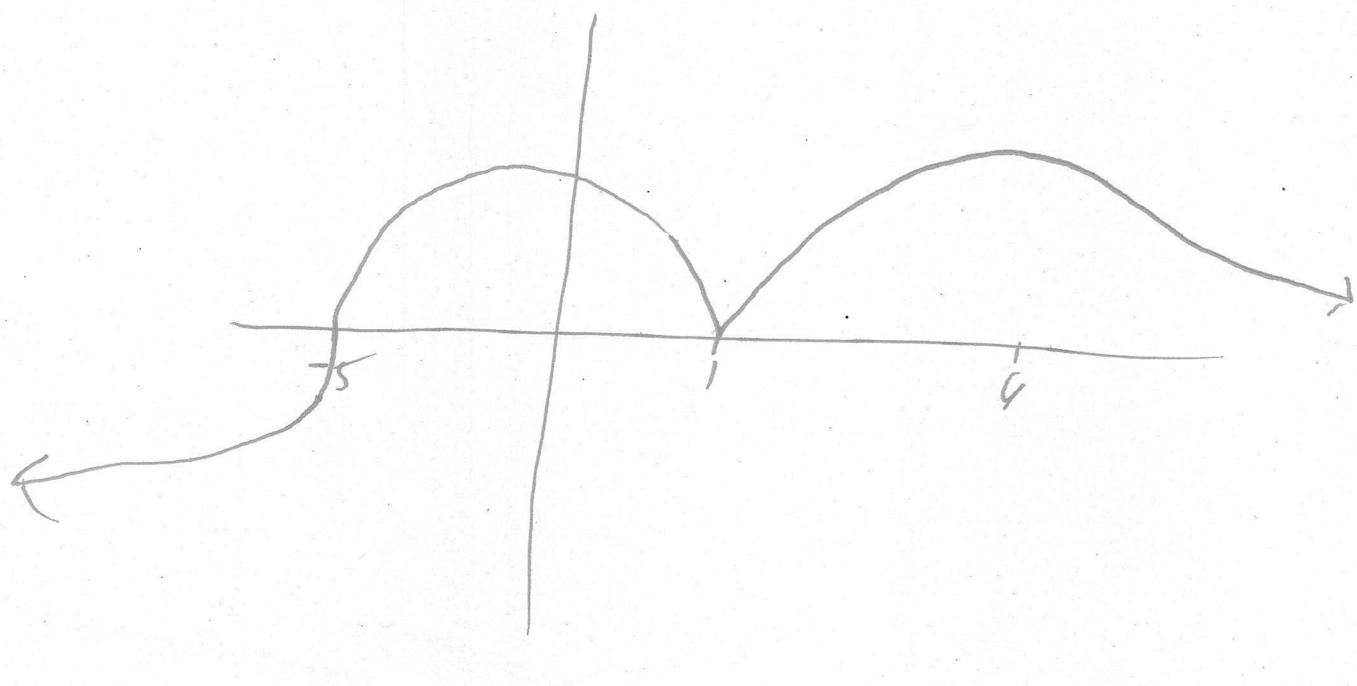
4. **(15 points)** a. Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x\to 3^+} f(x) = 2, \ \lim_{x\to 3^-} f(x) = 0, \quad f(3) = 1.$
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x\to\infty} f(x) = 4, \quad \lim_{x\to -\infty} f(x) = -1.$



b. Neatly sketch the graph of a single function $g(x)$ which has the following properties:

- $g(x)$ is continuous on $(-\infty, \infty)$
- $g'(6) = 0$
- $g(x)$ is not differentiable at $x = 1$
- $g(x)$ has a vertical tangent line at $x = -5$.



5. **(20 points)** Let $f(x) = 1/x$.

a. *Use the definition* of the derivative to prove that $f'(x) = -1/x^2$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{x-(x+h)}{h(x)(x+h)}$$

$$= \lim_{h\to 0} \frac{-h}{h\cdot x\cdot(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{\frac{-1}{x^2}}$$

b. Find the equation of the tangent line to $y = 1/x$ at the point where $x = 5$.

point $(5, 1/5)$ slope $= f'(5) = -1/25$

$$\boxed{y - 1/5 = -1/25(x-5)}$$

6. **(10 points)** The graph of a function $f(x)$ is given below. Use it to sketch the graph of the derivative $f'(x)$ on the same axes.