Name:

Math 141- Midterm Exam #1 - September 24, 2007

1. **(15 points)** True or false:

____ a. A function which is continuous at $x = a$ must also be differentiable at $x = a$.

____ b. It is possible for the graph of a function to have 3 vertical asymptotes.

____ c. The intermediate value theorem applies to $f(x) = 1/x$ on the interval $[-2, 1]$.

____ d. If $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$ then $\lim_{x\to 0}[f(x) - g(x)] = 0$

____ e. If $p(x)$ is a polynomial then $\lim_{x\to 5} p(x) = p(5)$.

2. **(20 points)**

a. Give the formal definition for $\lim_{x\to a} f(x) = L$.

b. Use the definition to prove that

$$\lim_{x\to 4} (3x - 7) = 5.$$

3. (**20 points**) Evaluate the following limits. If the limit does not exist then write DNE.

a. $\lim_{x \to -3} \frac{x^2-9}{x^2+2x-3}$

b. $\lim_{x \to 0} \frac{|x|}{x}$

c. $\lim_{x \to -\infty} \frac{\sqrt{x^2-9}}{2x-6}$.

d. $\lim_{x \to \infty} \frac{2x^2-5x+11}{x^2-2}$.

4. **(15 points)** a. Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x\to 3^+} f(x) = 2$, $\lim_{x\to 3^-} f(x) = 0$, $f(3) = 1$.
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x\to\infty} f(x) = 4$, $\lim_{x\to -\infty} f(x) = -1$.

b. Neatly sketch the graph of a single function $g(x)$ which has the following properties:

- $g(x)$ is continuous on $(-\infty, \infty)$
- $g'(6) = 0$
- $g(x)$ is not differentiable at $x = 1$
- $g(x)$ has a vertical tangent line at $x = -5$.

5. **(20 points)** Let $f(x) = 1/x$.

a. *Use the definition* of the derivative to prove that $f'(x) = -1/x^2$.

b. Find the equation of the tangent line to $y = 1/x$ at the point where $x = 5$.

6. **(10 points)** The graph of a function $f(x)$ is given below. Use it to sketch the graph of the derivative $f'(x)$ on the same axes.