Math 141A- Midterm Exam #3 - November 17, 2014

1. **(20 points)** A rectangle has its base on the x axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have and what are its dimensions?

2. **(10 points)** Suppose $f'(x) = \frac{1}{x} + \sin x$, for $x > 0$. Suppose also that $f(1) = 2$. Find $f(x)$.

3. **(10 points)** Find

$$\lim_{x \to 1^+} x^{1/(1-x)}.$$

4. **(15 points)** Let $f(x) = x^4 - 4x^3 + 10$. Find the critical values and classify them as local max, min, or neither using the **second derivative test**. If the second derivative test fails, you may use the first derivative test.

5. (**20 points**) Let

$$f(x) = \frac{(x+1)^2}{1+x^2}.$$

Using the quotient rule we obtain:

$$f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}, \quad f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}.$$

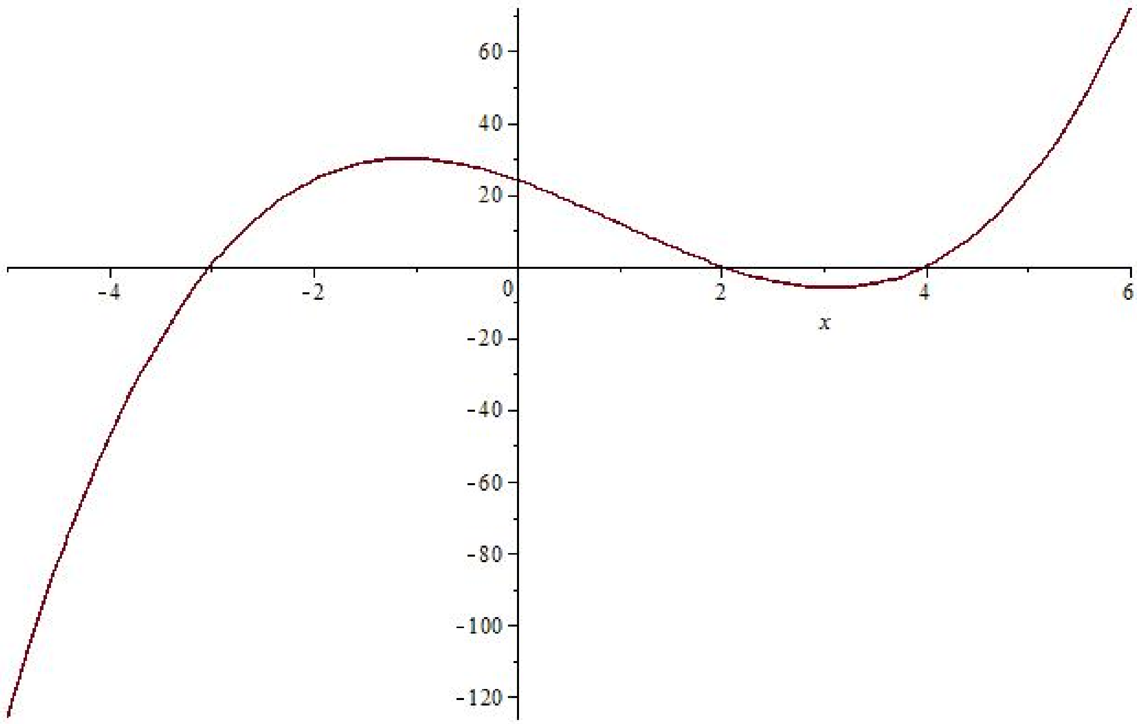
a. Find all x and y intercepts and any asymptotes.

b. Find the intervals where $f(x)$ is increasing or decreasing and any local maximums or local minimums.

c. Find the intervals where $f(x)$ is concave up or concave down, and determine any inflection points.

d. Neatly sketch the graph of $y = f(x)$, Label the x and y coordinates of any intercepts, local extrema and inflection points.

6. **(15 points)** Below is the graph of $y = f'(x)$. Find the intervals where the original function $f(x)$ is increasing/decreasing and concave up/down.



7. **(10 points)** Prove carefully that $x^4 + 3x + 1$ has exactly one root in the interval $[-2, -1]$. Make sure to cite any theorems that you use.

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