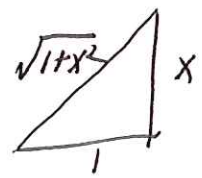
Lecture 13

Review
- inverse trig
- brief discussion of $\sec^{-1}, \csc^{-1}, \cot^{-1}$.
- Ex $\sin(\tan^{-1} x) = \dfrac{x}{\sqrt{1+x^2}}$



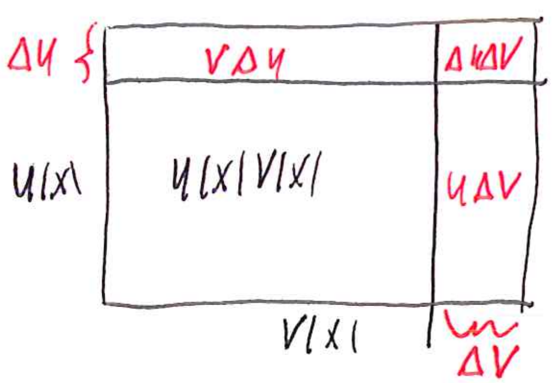
Back to Derivatives

So far:

$$\frac{d}{dx}(x^n) = nx^{n-1}, \quad \frac{d}{dx}(e^x) = e^x$$

$$(f \pm g)' = f' \pm g', \quad (cf)' = cf'.$$

Product Rule



Changing $u(x)$ by Δu and $v(x)$ by Δv changes $u(x)v(x)$ by $v\Delta u + u\Delta v + \Delta u \Delta v$.

So $\dfrac{\Delta(uv)}{\Delta x} = v\dfrac{\Delta u}{\Delta x} + u\dfrac{\Delta v}{\Delta x} + \Delta u \dfrac{\Delta v}{\Delta x}$ Now let $\Delta x \to 0$

Thm (Product Rule) Suppose $f(x)$ & $g(x)$ are differentiable. Then so is $f(x)g(x)$ and

$$\frac{d}{dx}\big(f(x)g(x)\big) = f(x)\frac{d}{dx}g(x) + \frac{d}{dx}f(x)\, g(x)$$

i.e. $(fg)' = fg' + f'g$

Proof
$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} g(x+h)\frac{(f(x+h)-f(x))}{h} + \lim_{h\to 0} f(x)\frac{(g(x+h)-g(x))}{h}$$

$$= g(x)f'(x) + f(x)g'(x) \quad //$$

Exs

1. $f(x) = x^2 = x \cdot x \qquad f'(x) = 1 \cdot x + x \cdot 1 = 2x$

2. $f(x) = x^2 e^x \qquad f'(x) = 2xe^x + x^2 e^x$

3. $f(x) = \sqrt{x}\, g(x) \qquad g(4) = 2 \quad g'(4) = 3 \qquad$ find $f'(4)$!

Quotient Rule Suppose $f(x), g(x)$ are diffble. Then so is $\frac{f(x)}{g(x)}$ when $g(x) \neq 0$, and

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}g(x)}{g(x)^2}.$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Ex $f(x) = \frac{1}{2x+1}$ $f'(x) = \frac{(2x+1)\cdot 0 - 1\cdot 2}{(2x+1)^2} = \frac{-2}{(2x+1)^2}$

Ex $f(x) = \frac{e^x+1}{x^2}$ Find tang line at $x=1$.

point $(1, 1+e)$

$f'(x) = \frac{x^2 \cdot e^x - (e^x+1)\cdot 2x}{x^4}$ $f'(1) = \frac{e - (1+e)\cdot 2}{1}$

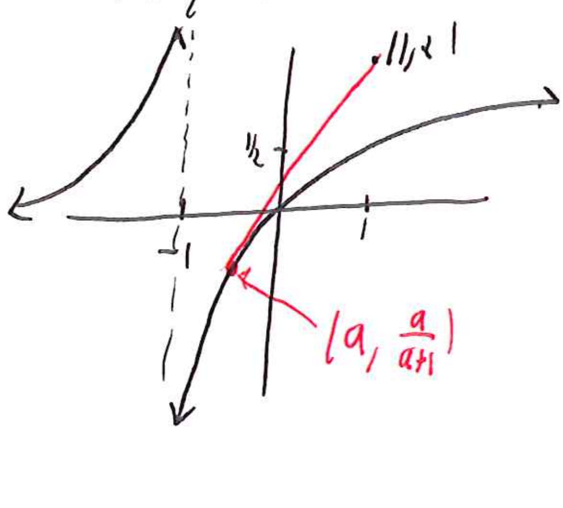
$= -2-e$

$y - 1 - e = (-2-e)(x-1)$

Ex $g(t) = \frac{4+t}{te^t}$ Find $g'(t)$

Ex How many tangent lines to curve ~~pi~~ $y = \frac{x}{x+1}$ pass through $(1,2)$? At what point do they touch curve?

$y' = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2}$



Tangent line $y - \frac{a}{a+1} = \frac{1}{(a+1)^2}(x-a)$. Is $(1,2)$ on it?

$$2 - \frac{a}{a+1} = \frac{1}{(a+1)^2}(1-a)$$

$$2(a+1)^2 - a(a+1) = 1-a \Rightarrow a = -2 \pm \sqrt{3}$$

Ex $f(4)=2$, $g(4)=5$, $f'(4)=6$, $g'(4)=-3$. Find $h'(4)$:

a. $h(x) = f(x)g(x)$

b. $h(x) = f(x)/g(x)$

c. $h(x) = 3f(x) + 8g(x)$