# Lecture 16

## Chain Rule

Review: Changing $x$ by small $\Delta x$ changes $f(x)$ by close to $f'(x)\Delta x$.

$y = f(u)$ $\quad u = g(x)$. $\quad$ Change $x$ by $\Delta x$ changes $u$ by $\frac{du}{dx}\Delta x$

Change $u$ by $\frac{du}{dx}\Delta x$ changes $y$ by $\frac{dy}{du}\frac{du}{dx}\Delta x$

Conclude

Changing $x$ by $\Delta x$ changes $y$ by $\frac{dy}{du}\frac{du}{dx}\Delta x$.

Thm (Chain Rule)

Suppose $g$ is diff'ble at $x$ and $f$ is diff'ble at $g(x)$. Then $F = f \circ g$ is diff'ble at $x$ and

$$F'(x) = f'(g(x))\,g'(x).$$

Leibniz notation: $y = f(u)$ $\quad u = g(x)$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Shorthand: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$

Example $y = \cos(x^2)$ $\quad f(x) = \cos(x)$ $\quad g(x) = x^2$

$y = f(g(x))$ $\quad f' = -\sin x$ $\quad g' = 2x$

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x) = -\sin(x^2) \cdot 2x$$

Example

$F(x) = (x^3+1)^{10}$ $\quad f(x) = x^{10}$ $\quad g(x) = x^3+1$ $\quad F = f(g(x))$

$F'(x) = 10(x^3+1)^9 \cdot 3x^2$

Remark Work from "outside in"

$F(x) = (-?-)^{10}$ $\quad F'(x) = 10(-?-)^9 \cdot \left(\text{derivative of what is inside}\right)$

Ex Differentiate $\sin(x^2)$ and $\sin^2 x$

Ex $g(t) = \sqrt{\dfrac{t^2+1}{t-7}}$ Find $g'(t)$

Ex $y = e^{\tan\theta}$. Find $\dfrac{dy}{d\theta}$.

Ex $F(t) = e^{t\sin 2t}$. Find $F'(t)$.

Ex $y = \sqrt{1-\sec x}$ Find $y'$ & $y''$

Ex

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 6 |
| 2 | 1 | 8 | 5 | 7 |
| 3 | 7 | 2 | 7 | 9 |

Let $h(x) = f \circ g$ Find $h'(1)$

$H(x) = g \circ f$ Find $H'(1)$

$F(x) = f(f(x))$ Find $F'(2)$

Ex $y = \sin(\sin x)$ Find tang line at $x = \pi$