141 Lecture 2

I. Average rate of change

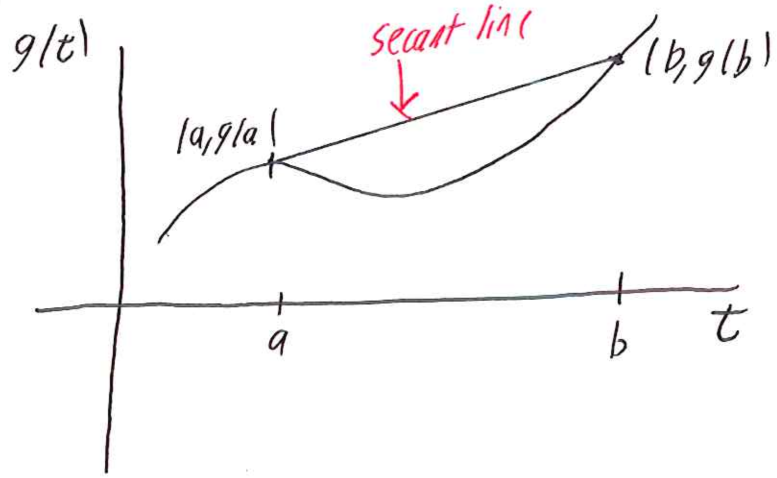
Fact Suppose $g(t)$ is a quantity depending on time t. The <u>average rate of change</u> of $g(t)$ from time $t=a$ to $t=b$ is

$$\frac{g(b)-g(a)}{b-a} \quad \begin{matrix}\leftarrow \text{Net change} \\ \leftarrow \text{time elapsed}\end{matrix}$$

Ex (last class) $f(t)=20t^2+10t$ miles in t hours

Avg speed $t=3$ to $t=5$ is $\frac{f(5)-f(3)}{5-3} = \frac{550-210}{2} = 170$ mph

Remark Average speed is the slope of a secant line



Problem $f(t)=20t^2+10t$. How fast at $t=3$?

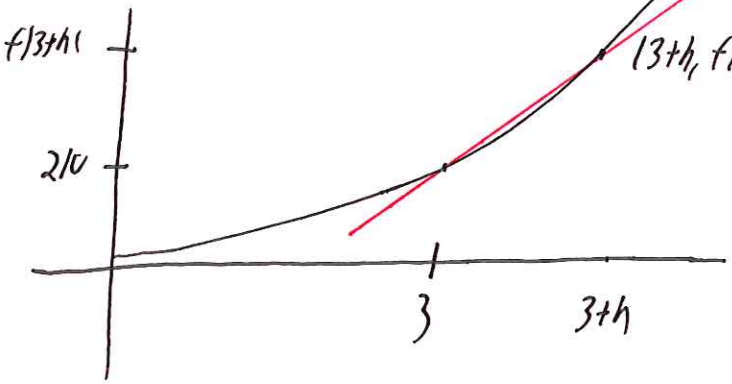
t	$f(t)$
2.9	197.2
2.99	208.702
3	210
3.01	211.3020
3.1	223.2

Avg speed $t=2.99$ to $t=3$ = $\frac{f(2.99)-f(3)}{2.99-3} = \frac{-1.298}{-.01} = 129.8$ mph

Avg speed $t=3$ to $t=3.01$

$\frac{f(3.01)-f(3)}{.01} = \frac{1.302}{.01} = 130.2$ mph

Key idea: Find avg. speed $t=3+h$ to $t=3$, let $h\to 0$.



Slope of secant = avg speed = $\dfrac{f(3+h)-f(3)}{3+h-3} = \dfrac{20(3+h)^2+10(3+h)-210}{h}$

$$= \frac{20h^2+120h+180+30+10h-210}{h} = \frac{20h^2+130h}{h} = 20h+130$$

So as $h\to 0$ conclude the instantaneous velocity at $t=3$ is

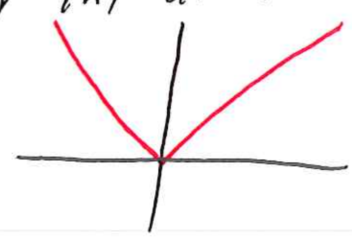
$$\lim_{h\to 0} 20h+130 = 130 \text{ mph}.$$

* Key idea: instantaneous rate of change of $f(t)$ at $t=a$ is slope of tangent line to graph $y=f(t)$ at $(a, f(a))$!

Arises as limit of secant lines.

Problems
1. When does it exist.
2. What is it?

Ex $y=|x|$ at $x=0$



II. Limits

Want to define $\lim_{x\to a} f(x) = L$.

"limit as x goes to a of $f(x)$ equals L"

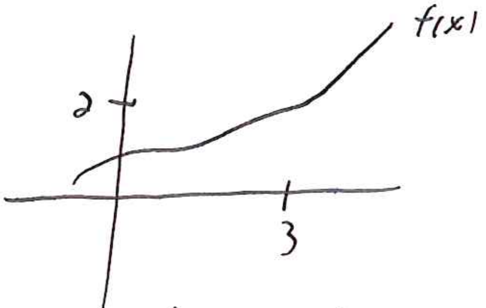
Maple $\lim_{x \to 0} \frac{\sin x}{x}$

Properties of limits

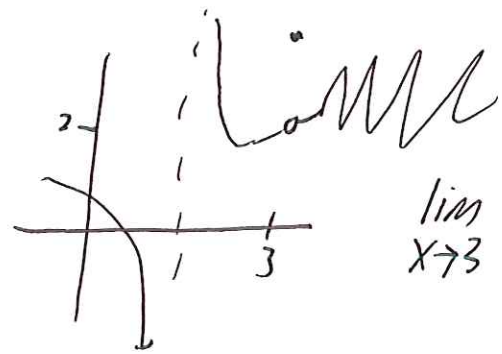
1. $f(a)$ is not relevant to determining $\lim_{x \to a} f(x)$. Indeed $f(a)$ may not exist.

2. Choose $\varepsilon > 0$ small. To calculate $\lim_{x \to a} f(x)$ it is enough to know values of $f(x)$ for x in $(a-\varepsilon, a+\varepsilon)$ $(a-\varepsilon < x < a+\varepsilon)$

Say $\lim_{x \to a} f(x)$ is local!



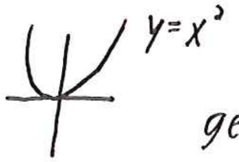
$\lim_{x \to 3} f(x) = 2$



$\lim_{x \to 3}$ still 2

Wrong Definitions

1. $\lim_{x \to a} f(x) = L$ means $f(x)$ gets "closer & closer" to L as x approaches a.

Ex 

gets closer & closer to -1 as $x \to 0$!

2. $f(x)$ gets as close as we like to L. Ex $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$

x	$f(x)$
1	0
.1	0
.01	0

Maple

III. Precise Definition of a limit

Informal: No matter how small a # (ε) I choose, if I confine myself to x values close enough (δ) to $x=a$ (but ignoring $x=a$), then $f(x)$ stays within ε of L.

Review
- $|b-a|$ measures distance on # line
- open interval

*** Def**

Let $f(x)$ be defined on some open interval containing $x=a$, but perhaps not at $x=a$.

Say $\lim_{x \to a} f(x) = L$ if:

For every $\varepsilon > 0$ there exists a $\delta > 0$ so that

If $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$