Lecture 22

Optimization

Goal: Find max or min values of $f(x)$, perhaps with some <u>constraints</u> on x.

<u>Def</u> Suppose $c \in D$ the domain of $f(x)$. Say

- $f(c)$ is an <u>absolute max</u> or <u>global max</u> value of $f(x)$ on D if $f(c) \geq f(x)$ $\forall x \in D$
- Similarly for absolute min

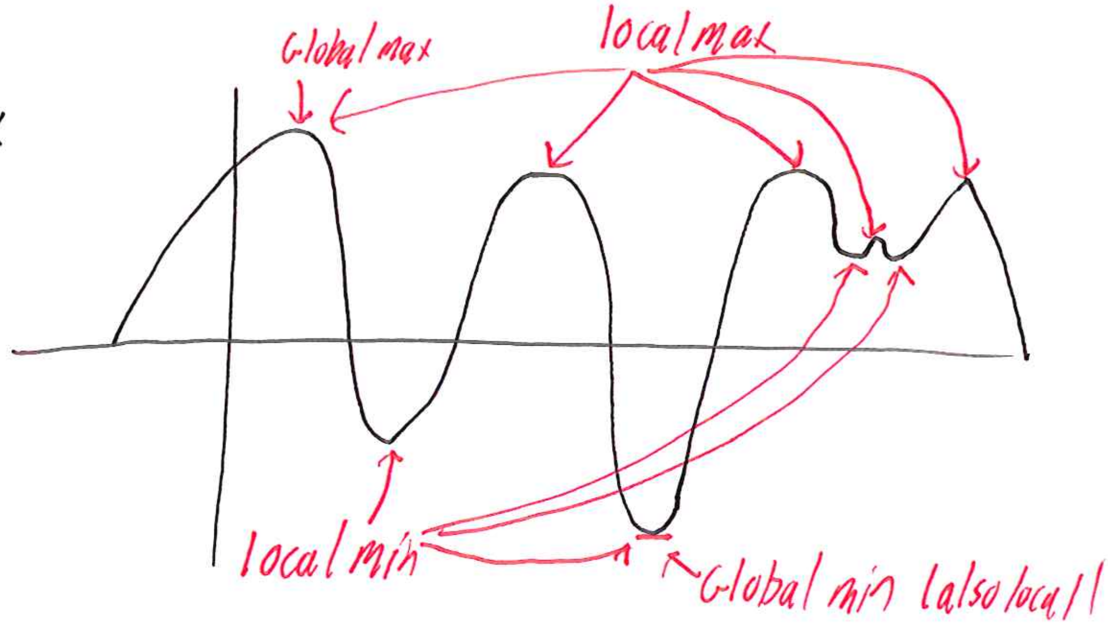
values called "extreme values"

<u>Def</u> $f(c)$ is a <u>local maximum</u> value if $f(c) \geq f(x)$ for all x in a neighborhood of c.

- Similarly for local min

<u>Recall</u> "neighborhood of c" means an open interval containing c.

Ex



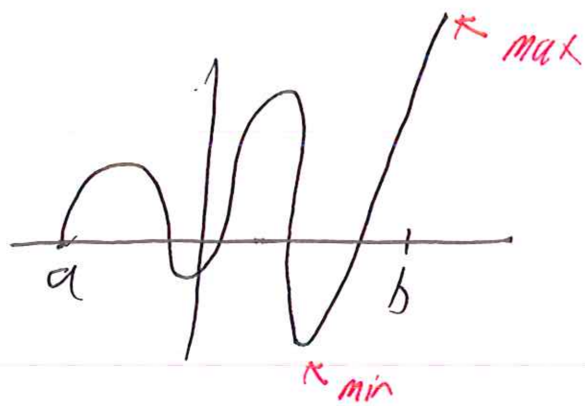
Ex $f(x)=x^2$ has global min value 0 at $x=0$
No max value.

Ex $y=\sin x$ has infinitely many global max values
at $\ldots -3\pi/2, \pi/2, 5\pi/2, \ldots$
min values at $\ldots -\pi/2, 3\pi/2, 7\pi/2, \ldots$

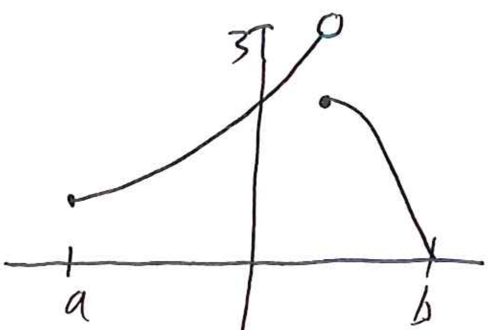
Extreme Value Thm

Suppose $f(x)$ is continuous on a closed interval $[a,b]$
Then $f(x)$ attains global max value and a global min value on $[a,b]$

Ex

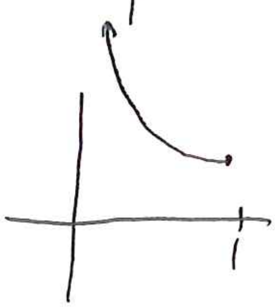


Ex



Does not attain max value,
Thm does not apply,
not continuous

Ex



$f(x)=1/x$, on $(0,1]$

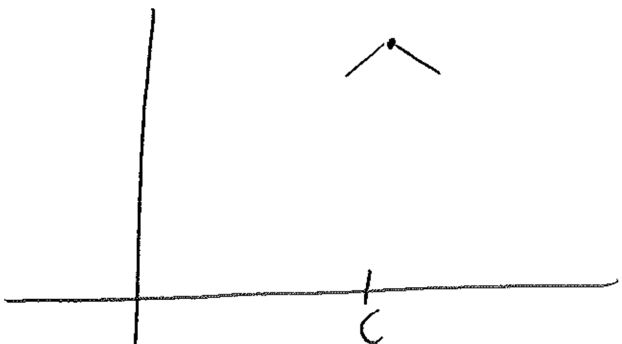
No max value, Thm does not apply, not closed interval

How to find max/min

<u>Fermat Thm</u> Suppose $f(x)$ has a local max or min at $x=c$.

Then either $f'(c)$ DNE or $f'(c)=0$.

<u>Proof</u> Suppose local max at $x=c$ and $f'(c)$ exists



For $h>0$ small

$f(c+h) \le f(c)$ so

$$\frac{f(c+h)-f(c)}{h} \le 0$$

For $h<0$ small

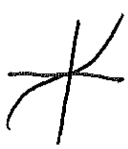
$f(c+h) \le f(c)$

so $$\frac{f(c+h)-f(c)}{h} \ge 0$$

Only possibility for $\displaystyle\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ is 0. //

<u>Ex</u> $f(x)=x^2$  min at 0

$=-x^2$  max at 0

$=x^3$  No local max/min