Lecture 23

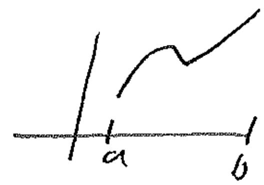
Extreme Value Thm Assume $f(x)$ is continuous on $[a,b]$. Then $f(x)$ attains global max value and global min value on $[a,b]$.

Q Which x values?

Fermat Thm Suppose $f(x)$ has a local max or min at $x=c$. Then:

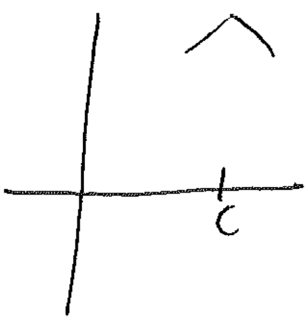
$$f'(c)=0 \quad \underline{\text{OR}} \quad f'(c) \text{ DNE.}$$

Rmk Endpoints of intervals are not local max/min.



Proof Suppose $f(x)$ has local max at $x=c$ and $f'(c)$ exists

Need: $f'(c)=0$



For $h>0$ $\quad f(c+h)\le f(c)$ so $\quad f(c+h)-f(c)\le 0$

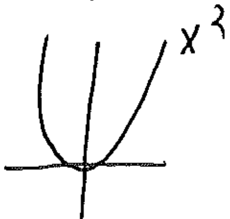
so $\quad \dfrac{f(c+h)-f(c)}{h} \le 0$

For $h<0$ $\quad f(c+h)\le f(c)$ so

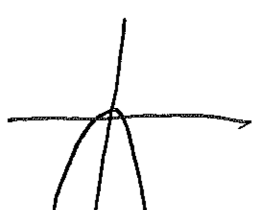
$\dfrac{f(c+h)-f(c)}{h} \ge 0.$

Only possibility for $\displaystyle\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ is 0.

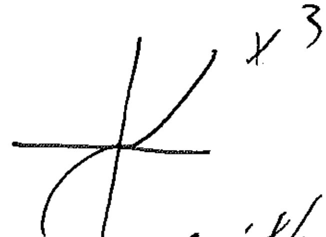
Remarks Anything is possible. $f(x) = x^2, -x^2, x^3$ all have $f'(0)=0$



local min



$-x^2$ local max



neither

Ex $f(x)=|x|$ has global min at $x=0$, $f'(0)$ DNE.

Def c is a critical # for f if $f'(c)=0$ or $f'(c)$ DNE, and $c \in$ Domain f

Finding max/min on closed interval $[a,b]$

1. Find crit values in $[a,b]$
2. Test endpoints a,b and C.V.'s
3. compare

Ex $f(x)=\ln(x^2+x+1)$. Find abs max & min on $[-1,1]$

$f'(x)=\dfrac{2x+1}{x^2+x+1}$ set $=0 \rightarrow x=-1/2$

x	$f(x)$	
-1	0	
$-1/2$	$\ln(3/4)$	⇐ global min
1	$\ln(3)$	⇐ global max

Ex find max & min values of $f(x)=x-2\tan^{-1}x$ on $[0,4]$

$f'(x)=1-\dfrac{2}{1+x^2}$

Set $=0 \rightarrow 1+x^2=2$

$x^2=1$

$x=\pm 1$

-1 not in int.

x	$f(x)$	
0	$0-2\tan^{-1}0=0$	⇐ min
1	$1-2\tan^{-1}1=3\pi/4$	⇐ max
4	$4-2\tan^{-1}4\approx 1.34$	

Ex $g(t)=\dfrac{t-1}{t^2-t+1}$ Find crit. #'s

$$g'(t)=\frac{t^2-t+1-\overbrace{(t-1)(2t-1)}^{(2t^2-3t+1)}}{(t^2-t+1)^2}=\frac{-t^2+2t}{(t^2-t+1)^2}$$

$g'(t)=0 \Rightarrow$ $\boxed{t=0 \quad t=2}$

Ex $f(x) = \frac{\ln x}{x^2}$. Find crit #'s.

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - 2x\ln x}{x^4} = \frac{x(1-2\ln x)}{x^4} = \frac{1-2\ln x}{x^3}$$

Set $f' = 0$ $\quad 1-2\ln x = 0$

$\ln x = 1/2 \quad \boxed{x = \sqrt{e}}$

Ex $f(x) = xe^{x/2}$. Find max/min on $[-3,1]$

$f'(x) = e^{x/2} + \frac{1}{2}xe^{x/2}$

set $= 0$ $\quad 0 = e^{x/2}\left(1 + \frac{x}{2}\right)$ $\quad \boxed{x = -2}$

never 0 $\quad x = -2$

Mean Value Thm

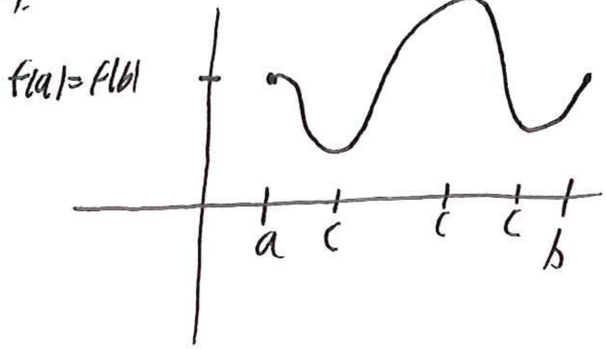
- Motivation
- special case

Rolles Thm Suppose:

1. f continuous on $[a,b]$
2. f is diff'ble on (a,b)
3. $f(a) = f(b)$

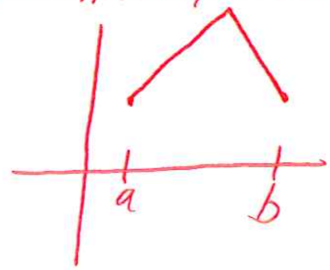
Then there is c in (a,b) with $f'(c) = 0$.

4.



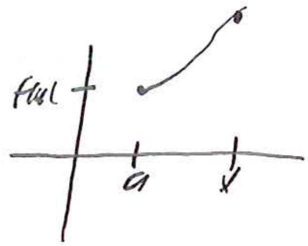
* may be multiple c

* differentiability required!



Proof

1. If $f(x)$ is constant then any c works



2. Suppose $f(x) > f(a)$ some x.

By Extreme value Thm

$\exists$ maximum in (a,b)

so local max so $f' = 0$ by Fermat

3. Suppose $f(x) < f(a)$ some x.

As above //

Ex Prove $x^3 + x - 1$ has exactly one real root.

Pf $f(0) = -1$, $f(1) = 1$ so by I.V.T a root exists in $(0,1)$

But $f'(x) = 3x^2 + 1 > 0$

2 roots + Rolles' would contradict this