Lecture 26

Review

1. $f''(x)$ tells concavity

2. 2^{nd} Der test: If $f'(c)=0$ then $f''(c)>0 \to$ local min, $f''(c)<0 \to$ local max

Ex $y=\dfrac{x^2}{x-1}$ $\quad y'=\dfrac{2x(x-1)-x^2}{(x-1)^2}=\dfrac{x^2-2x}{(x-1)^2}=\dfrac{x(x-2)}{(x-1)^2}$

f': $+$ | 0 | $-$ | 1 | $-$ | 2 | $+$

local max $x=0$
min $x=2$

$$y''=\frac{(x-1)^2(2x-2)-(x^2-2x)(2(x-1))}{(x-1)^2}$$

$f''(0)=\dfrac{-2}{1}$ local max

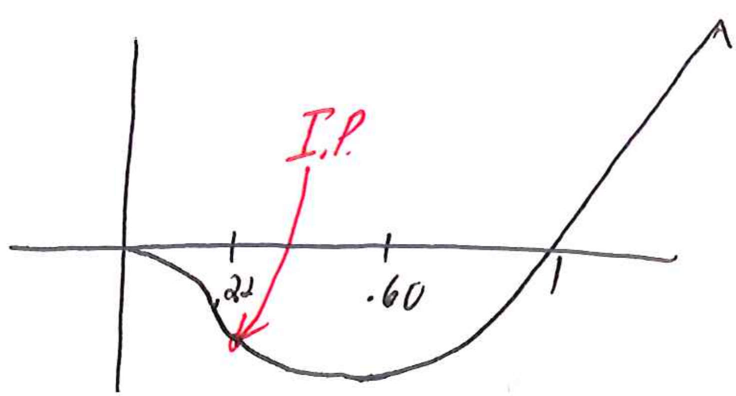
$f''(2)=\dfrac{2-0}{1}$ local min

Last time: $f(x)=x^2\ln x$

$f'(x)=x(2\ln x+1)$ $\quad$ $-$ | $1/\sqrt{e}\approx.60..$ | $+$

$f''(x)=2\ln x+3$ $\quad$ $-$ | $1/\sqrt{e^3}\approx.22$ | $+$

Later $\lim\limits_{x\to 0} x^2\ln x=0$



Ex

$f(x) = x^{1/3}(x+4)$ $f'(x) = \dfrac{4x+4}{3x^{2/3}}$ ⇐ vertical tang at $x=0$

f': $-$ on $(-\infty,-1)$, $+$ on $(-1,0)$, $+$ on $(0,\infty)$

Decr $(-\infty,-1)$

Inc $(-1,\infty)$

local min $(-1,-3)$

$f''(x) = \dfrac{\frac{4}{9}x - \frac{8}{9}}{x^{5/3}}$

f'': $+$ on $(-\infty,0)$, $-$ on $(0,2)$, $+$ on $(2,\infty)$

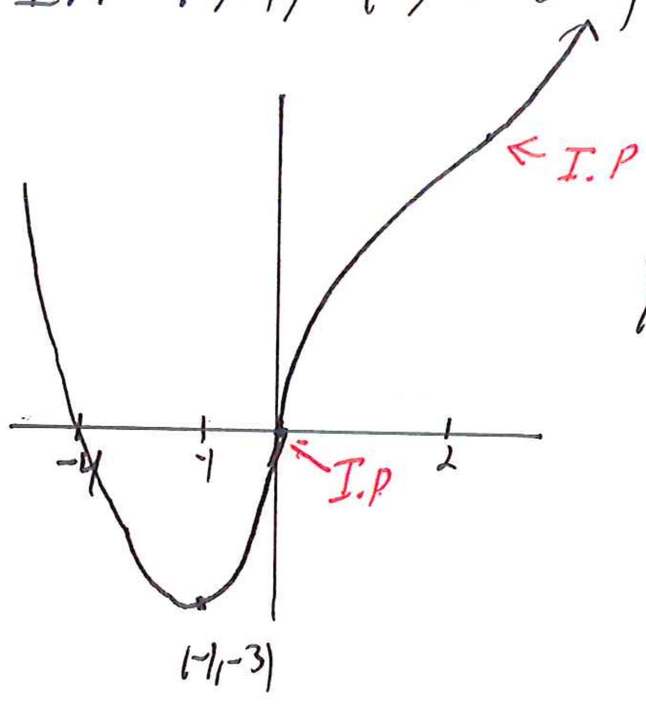
concave up $(-\infty,0) \cup (2,\infty)$

down $(0,2)$

I.P $(0,0)$, $(2, 6\sqrt[3]{2})$

intercepts $(-4,0)$

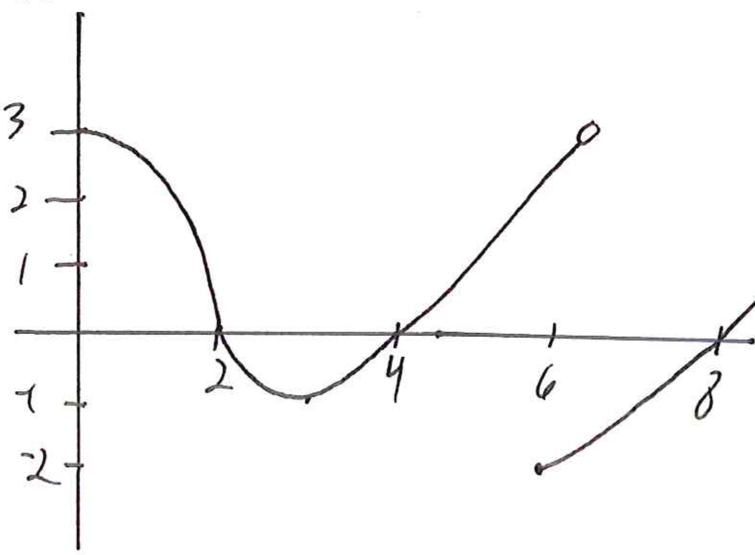
$(0,0)$



$y = x^{1/3}(x+4)$

4.3 # 35

Ex



Given graph of $f'(x)$.

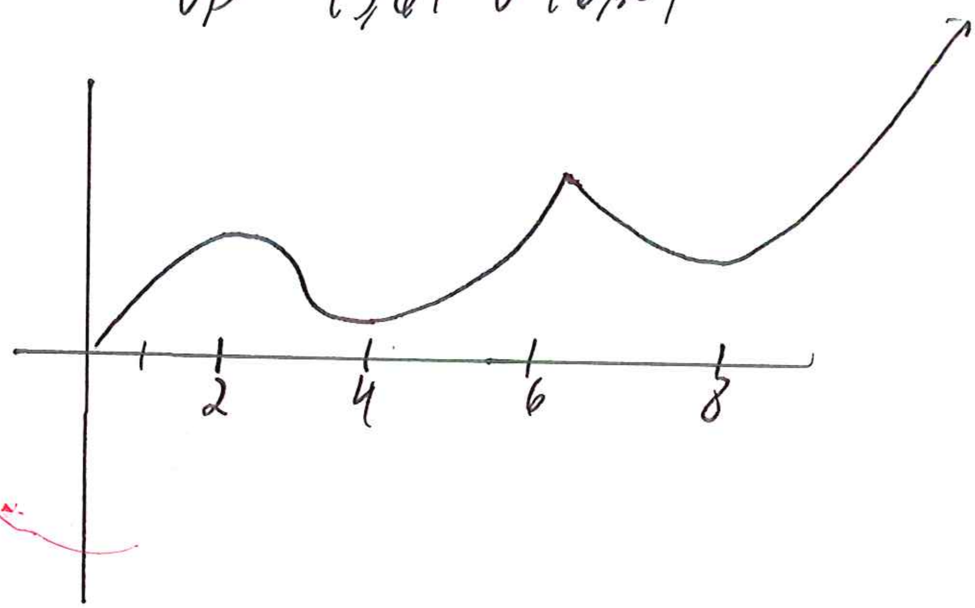
- Find int ↑, ↓, concavity, x coord of inf pts
- Assume $f(0) = 0$. Sketch $y = f(x)$

increasing $(0,2) \cup (4,6) \cup (8,\infty)$

decr $(2,4) \cup (6,8)$

concave down $(0,3)$

up $(3,6) \cup (6,\infty)$



Ex $f(x) = 1 + \frac{1}{x} - \frac{1}{x^2}$

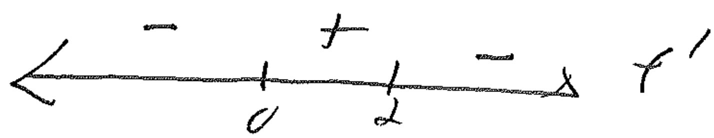
$\lim_{x \to 0^+} f(x) = -\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to \pm\infty} f = 1$

$f'(x) = -\frac{1}{x^2} + \frac{2}{x^3}$

Set $= 0$ $x^3 = 2x^2$ $x = 0, 2$



$f'(3) = -\frac{1}{9} + \frac{2}{27}$

$f'(1) = -1 + 2$

$f'(-1) = -1 - 2$

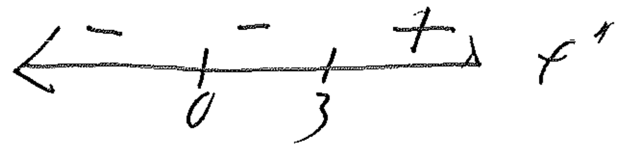
Decr $(-\infty, 0) \cup (2, \infty)$
Incr $(0, 2)$

Local max $(2, 5/4)$

$f'' = \frac{2}{x^3} - \frac{6}{x^4}$

set $= 0$ $2x^4 = 6x^3$ $x = 0$
$x = 3$

concave down $(-\infty, 0) \cup (0, 3)$
up $(3, \infty)$
I.P $(3, 11/9)$



$f''(4) = \frac{1}{32} - \frac{6}{256}$
$= \frac{1}{32} - \frac{3}{128} > 0$

$f''(1) = -4$

$f''(-1) = -2 - 6$