Lecture 35

Review FTOC Let $f(t)$ be continuous on $[a,b]$. Define

$g(x) = \int_a^x f(t)\,dt$. Then $g(x)$ is diffble and $g'(x) = f(x)$

$$\boxed{\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)}$$

* Relates integral & diff calculus
* Every continuous function has an antiderivative.

Maple example: $f(t) = \sin(\pi t^2/5)$

Part 2 Suppose $F'(x) = f(x)$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Problems

1. Find $\frac{d}{dx}\int_0^x \sqrt{t+t^3}\,dt$

2. Find $\frac{d}{dx}\int_x^0 \sqrt{3+\cos t}\,dt$

3. Find $h'(x)$, $h(x) = \int_1^{e^x} \ln t\,dt$

A $f(x) = \int_1^x \ln t\,dt$. Then $f'(x) = \ln x$

Now $h(x) = f(e^x)$ so $h'(x) = f'(e^x) \cdot e^x$

$= \ln(e^x)\, e^x = xe^x$.

4. $y = \int_{\sqrt{x}}^{\pi/4} \theta \tan\theta\, d\theta$. Find $\frac{dy}{dx}$

5 $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$ Find $g'(x)$

A: $g(x) = \int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_0^{3x} \_\_$

$= -\int_0^{2x} \frac{u^2-1}{u^2+1}\,du + \int_0^{3x} \frac{u^2-1}{u^2+1}\,du$ proceed as above

Definite integrals

1. $\int_0^4 2^5 \, ds$

2. $\int_1^2 x^3 + 2x^2 + 3$

3. $\int_0^4 \sqrt{x} \, dx$

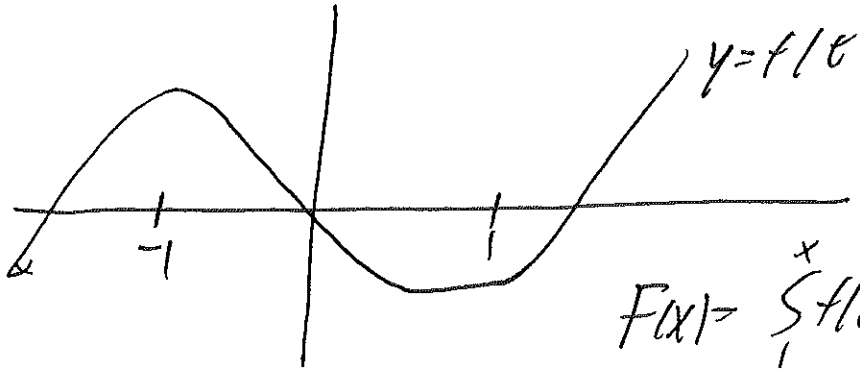
4. $\int_0^3 x \, dt = x \int_0^3 1 \, dt = x \cdot t \Big|_0^3 = 3x$

5. Calculate area enclosed by $y = 4 - x^2$ and $y = 0$.

6. Calculate area under $y = \sqrt[3]{x}$ $0 \le x \le 27$

7. At what values does Fresnel function have local max/min

8.



$F(x) = \int_1^x f(t) \, dt$.

Where is F concave down?

Indefinite integrals

Notation $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$.

Ex $\int \cos x\,dx = \sin x + C$

Use table on p. 403 except $\sinh$ & $\cosh$