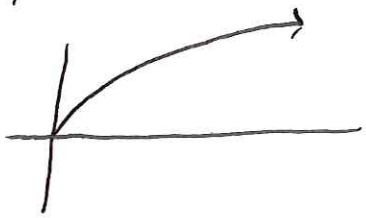
Lecture 6

Review $f(x)$ is **continuous at $x=a$** if $\lim_{x\to a} f(x) = f(a)$

Rmk Defined at a point, say "f is continuous" if continuous everywhere.

Ex $f(x) = \sqrt{x}$



Say $f(x)$ is **continuous from the right** at $x=a$ if $\lim_{x\to a^+} f(x) = f(a)$
" " " " " " **left** " " " " $\lim_{x\to a^-} f(x) = f(a)$

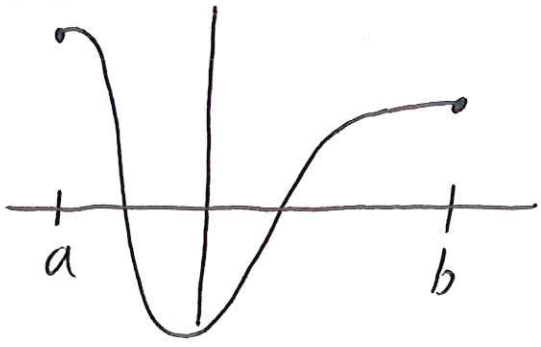
continuous at a $\longleftrightarrow$ cont. from right and left.

Ex $f(x) = \sqrt{x}$ is continuous on $[0,\infty)$

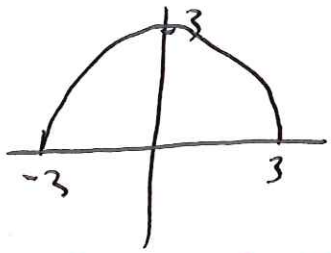
Def Say $f(x)$ is **continuous on closed interval** $[a,b]$ if $f(x)$ is:

- continuous on (a,b)
- cont from left at $x=b$
- " " right " $x=a$

Example



Ex $f(x) = \sqrt{9-x^2}$ cont on $[-3,3]$



* We will have several important theorems about cont functions on closed intervals

Limit laws $\Rightarrow$

Thm Suppose $f(x)$, $g(x)$ are continuous at a. So are $f \pm g$, f/g, $c \cdot f$

Compositions?

Ex $f(x) = x^2 + 3$ $\quad g(x) = \sin x$ $\quad f(g(x)) = \sin^2 x + 3$

$g(f(x)) = \sin(x^2 + 3)$

Thm If $f(x)$ is continuous at $x = b$ and $\lim_{x \to a} g(x) = b$ then

$$\lim_{x \to a} f(g(x)) = f(b)$$

Cor $g(x)$ continuous at a and $f(x)$ cont at $g(a)$

$\Rightarrow f \circ g$ cont at a.

Example $f(x) = \dfrac{x^3 + e^{\cos x}}{x^4 + 8}$ is continuous on $(-\infty, \infty)$

Ex $f(x) = \begin{cases} 1 - x^2 & x \le 1 \\ \ln x & x > 1 \end{cases}$ show f is continuous

Ex $f(x) = \begin{cases} x + 2 & x \le 0 \\ e^x & 0 \le x \le 1 \\ 2 - x & x > 1 \end{cases}$ where is f cont?

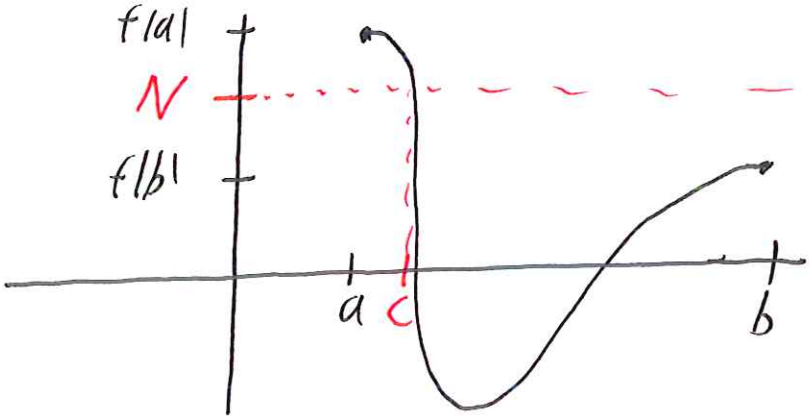
Ex $f(x) = \begin{cases} x^2 \sin(\pi/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$

Intermediate Value Thm

Suppose $f(x)$ is continuous on $[a,b]$

Suppose N lies between $f(a)$ and $f(b)$

Then there exists $c \in (a,b)$ so $f(c) = N$.



Rmk May be many choices for c.

Exs

1. Prove $x^4 + x - 3 = 0$ has a root in $(1,2)$
2. Hiking problem
3. Temp + Barom problem
4. Prove $\cos x = x^3$ has a solution
5. Examples not continuous