Math 353 Final Exam Review Sheet

The exam will be cumulative with extra emphasis on material done since the midterm, namely Chapters 8.1 and Chapters 11-15 and 17.1. Please note there are things we covered that are not in the book, if you missed class you should get the notes from someone. Also study your homework.

Definitions: You should know these definitions and be able to give examples illustrating them.

- Group, subgroup, cyclic group, subgroup generated by an element.
- Cayley table, symmetric group, dihedral group, abelian or nonabelian
- 1-line, 2-line, disjoint cycle notation
- Permutations avoiding a pattern (e.g. 231-avoiding from the HW)
- Symmetry group of an object.
- Isomorphism of groups.
- Cosets.
- Group acting on a set, transitive action.
- Orbit of an element, stabilizer of an element.
- Conjugacy class, centralizer of an element.
- Colorings, pattern inventory, weight function
- Cycle index of a group
- Rook polynomial of a board.
- Subboard of a board, disjoint subboards.

Theorems you should understand:

- Theorem 8.1 and its product version on the generating function for partition numbers. Also variations of it for subclasses of partitions, for example Lemma 8.2, Theorem 8.3 and its generalization on the homework.
- Definition of a generating function
- Cayley's Theorem and Lagrange's Theorem
- Orbit-Stabilizer Theorem (12.4) including understanding the proof
- The two theorems for counting orbits, including the more useful Frobenius Counting Theorem.
- Polya enumeration theory.
- Pigeonhole Principle, weak and strong version.
- Disjoint subboards theorem, "delete a square" theorem. You do not need to memorize the complementary board theorem but should understand how we proved it using I/E principle
- Rook polynomials are invariant under permutations of the rows or columns of a board.

Things you should be able to do.

- Multiply elements of the symmetric group and put the answer in disjoint cycle notation
- Calculate the order of elements of the symmetric group. Calculate the number of elements in a conjugacy class of the symmetric group (i.e. elements with a fixed cycle type).
- Given a group action, determine orbits and stabilizers.
- Prove certain subsets are subgroups.
- Apply Frobenius' Theorem to count orbits of a group action (exercises in 13.2)
- Apply Polya Counting Theorem.
- Calculate the rook polynomial of a board, applying the appropriate theorems when necessary.
- Solve basic problems using the Pigeonhole principle.
- Given a group and a subgroup, determine the cosets of the subgroup.
- Calculate generating functions for a sequence given a linear recurrence.
- Manipulate generating functions to prove things.

*Review all homework solutions and quizzes.