**Math 353 Homework #10- Due Tuesday 11/22/16**

1. 13.2.1B

2. 13.2.2B (Hint: The symmetries of the object in Figure 13.5 are the same as the symmetries of a square)

3. 13.2.4B

4. Suppose we want to place 4 red, two yellow and two green keys on a circular key ring. Use Burnside's Theorem to count the number of ways to do this.

5. Find the number of different colorings of a cube with two white, one black and three red faces.

6. How many different chemical compounds can be made by attaching $H$, $CH_3$, $C_2H_5$ or $Cl$ radicals to the four bonds of a carbon atom. (The radicals lie at the vertices of a regular tetrahedron with the carbon atom in the center).

7. Give a simple proof of Cauchy's theorem for $p = 2$. (Hint: pair up)

8. Suppose $H$ is a subgroup of $G$ and $g \in G$. Let:

$$gHg^{-1} = \{ghg^{-1} \mid h \in H.\}$$

a. Prove that $gHg^{-1}$ is also a subgroup.

b. Let $X$ be the set of all subgroups of $G$. Prove that $G$ acts on $X$ by conjugation, as in part a.

c. The stabilizer of a subgroup $H$ under this action is called the *n*ormalizer:

$$N_G(H) = \{g \in G \mid gHg^{-1} = H.\}$$

Let $G = S_4$ and $H = \langle(1, 2, 3, 4)\rangle$ be a cyclic subgroup of order 4. Determine the normalizer of $H$.