Math 353 Homework #3- Due Wednesday 9/21/16

1. 3.1.5B

2. 3.3.2B

3. The total number B_n of partitions of an n element set (equivalence relations on an n element set) is the sum of Stirling numbers of the second kind, namely:

$$B_n = \sum_{k=0}^{n} S(n, k).$$

The numbers B_n are called the Bell numbers, after E.T. Bell. Use the recursion formula from 3.3.2B to get the following:

$$B_n = \sum_{j=0}^{n-1} \binom{n-1}{j} B_j.$$

4. Give a combinatorial proof of the result in Problem 3. Hint: Pick a fixed element in the n element set and sort by how many elements are in its equivalence class.

5. A store sells eight different kinds of candy. In how many ways can you choose a bag of 15 pieces?

6. In how many ways can n identical chemistry books, r identical mathematics books, s identical physics books and t identical astronomy books be arranged on three distinct bookshelves. Assume there is no limit on the number of books per shelf.

7. In how many ways can 12 distinct pieces of candy be placed in four identical bags so that each bag has at least one piece? Repeat the problem except replace the 4 bags with 4 distinct children.

8. Write down the value of the Bell numbers $B_1, B_2, \ldots, B_8$. Observe the ratio of each to the proceeding. Can you make any intuitive statement about how fast B_n grows as a function of n. Notice that B_2, B_3, B_7 are prime. It is an open problem if there are infinitely many prime Bell numbers.