Math 353 Homework #9- Due Friday 11/11/16

1. 12.2.2B

2. Let H and K be two subgroups of a group G. Prove that their intersection $H \cap K$ is also a subgroup. For extra credit prove that the union $H \cup K$ is never a subgroup except in the trivial situation where $H \subseteq K$ or $K \subseteq H$.

3. Let G be a group and $g \in G$. Define the *centralizer* of g, denoted $C_G(g)$, as the elements that commute with g,namely:

$$C_G(g) = \{x \in G \mid xg = gx\}.$$

a. Prove that $C_G(g)$ is a subgroup of G.
b. Let $\sigma = (1,2)(3,4) \in S_4$ Calculate $C_{S_4}(\sigma)$.
c. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in GL_2(\mathbb{Q})$. Calculate the centralizer of A.
d. Describe the center $Z(G)$ in terms of centralizers.

4. Calculate the conjugacy classes in the dihedral group D_8. Repeat for D_{10}.

5. 12.3.2A

6. 12.4.1B

7. Let $G = S_4$ be the symmetric group on 4 letters. Let $H = \{e, (12)(34), (13)(24), (14)(23)\}$ and let $K = \{e, (12), (34), (12)(34)\}$. Verify that H and K are both subgroups of S_4 and both are isomorphic to the Klein 4 group. Next compute the left and right cosets of H. Repeat for K. What do you notice? .