Name:

Math 353 Midterm Exam - October 17, 2014

Instructions: You may not use any notes, books, calculators, etc... It is ok if your final answers include binomial coefficients and if you do not multiply out exponentials.

1. **(55 points)** Short answer, little or partial credit.

a. Define a *derangement*.

b. A class of 20 students wishes to elect a president, vice president, and three senators. How many ways can this be done?

c. A five-card poker hand is dealt. What is the probability of getting two-pair? (for example AATT3 is two-pair but AAATT is not, it is a full-house.)

d. Let $\sigma = (123)(45)(678)$ and $\tau = (1568)(234)(7)$ be elements of the symmetric group S_8.. Write the products $\sigma\tau$ and σ^2 in cycle notation.

e. Let $S(n, k)$ denote the Stirling number of the second kind. Calculate $S(5, 2)$.

f. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 15$ where the x_i are nonnegative integers?

g. How many triangulations are there of a octagon?

h. Calculate $p(8, 3)$, the number of partitions of 8 with less than or equal to three parts.

i. A fair coin is tossed 6 times. What is the probability of getting at least 4 heads?

j. A biased coin comes up heads with probability 0.8. If it is tossed 6 times what is the probability of getting at least 4 heads.

k. Find the generating function for the sequence $a_n = n$.

2. **(15 points)** A bag contains 3 red, 2 blue and 6 green marbles. A marble is sampled and replaced 7 times. Find the probability that all three colors have already been seen.

3. **(15 points)** a. Define the Stirling number of the second kind, $S(n,k)$.

b. Give a combinatorial proof of the identity: $S(n+1,k+1) = \sum_{i=k}^{n} \binom{n}{i} S(i,k)$.

4. **(15 points)** Consider the sequence given by $a_1 = 8, a_2 = 46$ and $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 3$. Solve the recursion to find a formula for a_n.