Review of some groups to know for Exam 1.

Cyclic group of order n: Written additively usually denoted $\mathbb{Z}/n\mathbb{Z}$ or $\mathbb{Z}_n$, it is $\{0, 1, 2, \ldots n-1\}$ under addition mod n. Multiplicatively it can be written as $\{1, x, x^2, \ldots, x^{n-1} \mid x^n = 1\}$ or as the n-th roots of unity $\{e^{\frac{2k\pi i}{n}} \mid 0 \leq k \leq n-1\}$. You should know the theorems telling us all subgroups of this group, and which subgroup is generated by which elements. In particular which elements generate the entire group.

Infinite cyclic group: We proved that every infinite cyclic group is isomorphic to the integers under addition.

Klein 4-group V: This is the four element abelian group $\{1, x, y, xy\}$ with every element squaring to give 1.

U(n): The group of units mod n. This is the numbers from $1, 2, \ldots n-1$ which are relatively prime to n with operation multiplication mod n. Thus it has $\phi(n)$ elements and is abelian.

Symmetric group S_n. This is the permutations of the set $\{1, 2, \ldots, n\}$. It is a group under composition of functions. Its order is $n!$ You should know how to write down the elements in this group in disjoint cycle notation and multiply them and determine their orders.

Alternating group A_n. This is the subgroup of S_n consisting of the *even* permutations, i.e. those which can be written as a product of an even number of transpositions. It has order $n!/2$. We know for $n > 2$ that it can be generated by the 3-cycles in S_n.

Symmetry groups: The group of symmetries of a planar object are the set of bijections from the plane to itself which preserve distance and preserve the object. They form a group under composition of functions. These groups may be finite or infinite, depending on the object.

Dihedral group D_n: This is the symmetries of a regular n-gon. It has order $2n$, with n rotations and n reflections. If r is a rotation by $360/n$ degrees then the elements in the group are

$$\{e, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$$

and the entire Cayley table can be determined by the relations $r^n = s^2 = e$ and $rs = sr^{-1}$.

General linear group $GL_n(F)$: This is invertible $n \times n$ matrices with entries in the field F. If F is a finite field, then this is a finite group, otherwise it is infinite.

Aut(G): Given any group, the set of automorphisms of the group form a new group under composition of functions, called the automorphism group of G..

Some subgroups:

Trivial subgroups: The identity $\{e\}$ and the entire group G are always subgroups of G.

Subgroup generated by an element: For any $g \in G$, $< g >$ is the cyclic subgroup generated by G. This subgroup has order equal to the order of g.

Center $Z(G)$: The center is the elements of G which commute with all elements of G. We have $G = Z(G)$ if and only if G is abelian. It is possible for the center to be only $\{e\}$.

Centralizers: For each $g \in G$ there is a subgroup, the centralizer of g

$$C(g) = \{x \in G \mid xg = gx\}.$$

It is always true that $< g > \leq C(g)$ and it is trivially true that the center is the intersection of all the centralizers.