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4. $s = -3, t = 2$ works as does $s = 8, t = -5$ etc...

7. See back.

24. Assume by induction Euclid's lemma holds for products of $n-1$ elements. The original lemma is the base case of 2 elements. How suppose $p|a_1 a_2 \cdots a_n$. Then $p|a_1 a_2 \cdots a_{n-1}$ or $p|a_n$ by the original lemma. If $p|a_n$ we are done. Otherwise $p|a_1 a_2 \cdots a_{n-1}$ in which case p divides one of $a_1, a_2, \ldots, a_{n-1}$ by our inductive assumption.

28. Notice f_1 and f_2 satisfy $f_n < 2^n$. Assume that $f_t < 2^t$ for all $t < n$. Then

$$\begin{aligned} f_n &= f_{n-1} + f_{n-2} \\ &< 2^{n-1} + 2^{n-2} \\ &< 2^{n-1} + 2^{n-1} = 2^n \end{aligned}$$

Thus $f_n < 2^n$ as desired.

48. $a - a = 0$ so $a \sim a$. If $a - b$ is an integer so is $-(a-b) = b - a$ so $\sim$ is reflexive. Finally if $a - b$ and $b - c$ are integers, so is $(a-b) + (b-c) = a - c$ so $\sim$ is transitive. A complete set of equivalence class representatives is the set of real number in $[0, 1)$.

50. $a + a$ is even so $\sim$ is reflexive. If $a + b$ is even, so of course is $b + a$ so $\sim$ is symmetric. Finally suppose $a+b$ is even and $b+c$ is even. Then so is $a+b+b+c = a+2b+c$. But $2b$ is also even so $a+2b+c-2b = a+c$ is even, so $a \sim c$ and $\sim$ is transitive. There are two equivalence classes, one is all the odd integers, the other is all the even integers.

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2.

	e	r	r^2	s	sr	sr^2
e	e	r	r^2	s	sr	sr^2
r	r	r^2	e	sr^2	s	sr
r^2	r^2	e	r	sr	sr^2	s
s	s	sr	sr^2	e	r	r^2
sr	sr	sr^2	s	r^2	e	r
sr^2	sr^2	s	sr	r	r^2	e

3. No

5. See back

11. See back

13. Let H and V be the horizontal and vertical flips. The elements are e, H, V and HV. Every element squares to the identity, the group is abelian . The Cayley table is easy from that point!

16. The symmetries are flips across infinitely many vertical lines, also a single horizontal flip and a shift an integer number of units in either direction. It is not abelian.

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3. Notice $2 \cdot 2 = 4$ which is congruent to 0 mod 4. Thus the set $\{1, 2, 3\}$ is not closed under multiplication mod 4. The set $\{1, 2, 3, 4\}$ is a group under multiplication mod 5. It is closed. The identity is 1, The inverse of 2 is 3. 1 and 4 are their own inverses.

4. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ do not commute. (there are of course infinitely many other possible examples!)

28. The identity is $1 = 3^0 6^0$. Multiplication of rationals is associative. The set is closed under multiplication. Finally notice that the inverse if $3^m 6^n$ is $3^{-m} 6^{-n}$.