Page 132.

18. We know Aut($\mathbb{Z}_{50}$) is isomorphic to $U(50)$ and the automorphisms are given by multiplication by elements of $U(50)$. Trial and error (or the Euclidean algorithm) tells us that $\phi(7) = 13 \equiv 63$. Thus $\phi(x) = 9x$

22. They are not isomorphic. In $U(24)$ every element squares to the identity, not so in $U(20)$.

30. Suppose G is a finite abelian group with no element of order 2. Define $f(g) = g^2$. Since G is abelian we have $f(xy) = (xy)^2 = x^2y^2 = f(x)f(y)$, so f is a homomorphism. Suppose $f(x) = f(y)$. Then $x^2y^2 = e$. Again using the abelian assumption we can rewrite this as $(xy^{-1})^2 = e$ which means $xy^{-1} = e$ since no element has order two. Thus $x = y$ and f is one-to-one. Since G is finite f must also be onto, so it is an automorphism. Now lot $G = \mathbb{Q}^*$. Notice f is not onto, for instance 2 is not in the image since $\sqrt{2}$ is not rational.

31. See back.

43. Let f be an automorphism of $\mathbb{R}^*$. Let $a > 0$. Then $a = \sqrt{a}\sqrt{a}$ so $f(a) = (f(\sqrt{a}))^2 > 0$, so f maps positive numbers to positive numbers. Now let $b < 0$. Suppose $c = f(b) > 0$. Recall that f^{-1} is also an automorphism, so $f^{-1}(c) = b < 0$ which contradicts what we just showed. Hence f takes negative numbers to negative numbers.

13. See back.

29. Yes, see back.

35. As shown in the back, If $|a| = n$ then $(\phi_a)^n = id$ so the order of ϕ_a must divide n. Notice that $(\phi_a)^n = \phi_{a^n}$ and this is the identity map if and only if $a^n \in Z(G)$. Thus the order of ϕ_a is the smallest power of a which lies in the center $Z(G)$. In D_4, r has order 4 but $r^2 \in Z(D_4)$ so ϕ_r has order 2.

42. Let $\phi : \mathbb{Q} \to H \leq \mathbb{Q}$ be an isomorphism. We must show $H = Q$. Suppose $\phi(1) = x$. Then $phi(a) = ax$ is immediate. Plugging the equation $1/b + 1/b + \cdots + 1/b = 1$ into ϕ we get $\phi(1/b) = x/b$. Together we get $\phi(a/b) = ax/b$. Since $\phi(0) = 0$ we know $x \neq 0$. However all rational multiples of x are in the image, so ϕ maps onto $\mathbb{Q}$ as desired.

31. See back.

33. See back.

34. $\phi_h = \phi_g$ if and only if $g^{-1}h \in Z(G)$.

39. See back. also done in class.

Page 148.

1. See back.

2. There are 6 since S_4 has order 24 and H has order 4.

4. The condition becomes $-a + b \in H$, or since G is abelian, $b - a \in H$.

5. The only divisors of pq are $1, p, q$ and pq. Thus any proper nontrivial subgroup has prime order, and hence is cyclic by Cor 3 page 141.