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8. Let $H = \langle a^5 \rangle \leq \langle a \rangle$. Since a^5 has order 3, the index is five so we expect 5 left cosets. They are $\{H, aH, a^2H, a^3H, a^4H$.

12. For $z = a + bi$ recall that the norm of z is $|z| = a^2 + b^2$ and that $|z_1 z_2| = |z_1||z_2|$. The subgroup H is all numbers of norm 1. Then $z_1H = z_2H$ if and only if $z_2 z_1^{-1} \in H$ if and only if $|z_1| = |z_2|$. Thus the cosets of H are just circles centered at the origin of different radii.

16. Let a be relatively prime to n. Then $a \in U(n)$ and $U(n)$ has order $\phi(n)$. Thus by Corollary 4, $a^{\phi(n)} = e$ in the group $U(n)$. However $U(n)$ is a group under multiplication mod n so this just means $a^{\phi(n)} \equiv 1 \bmod n$.

20. Since $H \cap K$ is a subgroup of both H and K, by Lagrange its order must divide both orders. In this case that forces the order to be 1.

31. See back.

33. See back.

39. See back.

40. The orbit of the point Q will be the circle centered at P containing Q.

44. a. 12
b. 24
c. 60
d. 60

46. Theorem 7.3/ Example 9 tell us that the rotation group of the soccer ball has 60 elements. Now suppose it had a 60 degree rotational symmetry as described. This would give an element of order 6 in the stabilizer of a hexagon. But we know the orbit of the hexagon has 12 elements, which would mean the rotation group has at least 6*12=72 elements, a contradiction.