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11. $Z_4 \oplus Z_4$ has one element of order 1, 3 elements of order 2 and 12 elements of order 4. See back for explanation of rest.

12.If $D_n$ were the direct product of $< r >$ and $Z_2$ then it would be abelian since both $< r >$ and $Z_2$ are.

20. $S_3 \oplus Z_2$ is not abelian, ruling out the first two cases. It has only 2 elements of order 3, and $A_4$ has eight, ruling out $A_4$. So it is $D_6$.

42/43. Any isomorphism $\phi$ from $Z_12$ to $Z_4 \oplus Z_3$ is uniquely determined by $\phi(1)$, we must choose $\phi(1)$ of order 12. Thus we can set $\phi(1)$ to be $(1,1), (3,1), (1,2)$ or $(3,2)$. I.e. if $\phi(1) = (3,1)$ then $\phi(2) = (6,2) = (2,2)$, etc.. Thus there are 4 isomorphisms.

58. $U(144) = U(16) \oplus U(9)$ (by 8.3). But $U(16) = Z_2 \oplus Z_4$ and $U(9) = Z_6$ by p.158. Thus

$$U(144) \cong Z_2 \oplus Z_4 \oplus Z_6.$$

Similarly $U(140) = U(4) \oplus U(5) \oplus U(7) \cong Z_2 \oplus Z(4) \oplus Z(6)$ so they are isomorphic.

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1. No, $(1,3)H \neq H(1,3)$.

4. No, H is not normal. For example if $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ then $AH \neq HA$.

8. $(1,2,3)(1,2)(3,4)(1,2,3)^{-1} = (2,3)(1,4)$ is not in $H$ so $H$ is not normal. One can see from the multiplication table that behavior described. This shows the left multiplication of cosets is not well-defined.

11. No! See class notes, $V \trianglelefteq A_4$ and $A_4/V = Z_3$ but $A_4$ is not abelian.

16. The order of $gH$ in $G/H$ is the smallest power of $g$ which lies in $H$. In this example that order is 3.

26. Notice how every element in $G$ squared lies in $H$. Thus all element of $G/H$ have order 1 or 2 so $G/H$ is $Z_2 \oplus Z_2$.

38. If $aH$ has order 3 then $a^3$ is in $H$. Elements of $H$ have order $1, 2, 5$, or 10. Thus $a$ has order $3, 6, 15$ or 30.

42. If $H$ is the only subgroup of its order in $G$ then it must be normal. This is because $gHg^{-1}$ is also a subgroup of $G$ of the same order, and hence must

equal $H$ or any $g \in G$.

49. See back.

56. a. Notice that

$$gx^{-1}y^{-1}xyg^{-1} = (gxg^{-1})(gyg^{-1})(gxg^{-1})^{-1}(gyg^{-1})^{-1}.$$

Thus conjugating any of the generators gives another generator, so the group is normal.

b. Since $xyx^{-1}y^{-1}$ is always in $G'$ the rule for equality of cosets tell us that $xyG' = yxG'$, i.e. $xG'yG' = yG'xG'$ so $G/G'$ is abelian.

c. If $xNyN = yNxN$ for all $x, y$ then $xyx^{-1}y^{-1} \in N$ for all $x, y$ so $N \leq G'$.

d.

58.If $H$ has order $n$ then any $gHg^{-1}$ is another subgroup of order $n$. Also if $H$ has order $n$ then $H = g(g^{-1}Hg)g^{-1}$ so $H$ is of the form $gPg^{-1}$ where $P$ has order $n$. Thus if $N$ is the intersection of all subgroup of order $N$, then $gNg^{-1}$ is the same intersection, so equal to $N$. Thus $N$ is normal.

65. We know $G/Z$ has order 6 and cannot be cyclic, so it must be $D_6$.