**Homework 2**
due of first class of week 5

**Problem 1: ( unconstrained optimization ) ( 3 points)**
Find the values of x and y that optimize the value of the following function f. Find this optimal value. Is it a maximum or a minimum?

\[ f(x, y) = 4x^2 - 18xy + 21y^2 - 4x + 6y + 20 \]

**Solution:**
\[ f_x = 8x - 18y - 4, \quad f_y = -18x + 42y + 6 \]
Solve \( f_x = f_y = 0 \) to get \( x = 5 \) and \( y = 2 \)

Test \( D = f_{xx}f_{yy} - (f_{xy})^2 = 8 \cdot 42 - (-18)^2 = 12 > 0 \), and \( f_{xx} > 0 \), therefore the function has a minimum at \((5,2)\). The minimum value is \( f(5,2) = 16 \)

**Problem 2: ( constrained optimization ) ( 4 points)**
Three thousand dollars are funded to fence off a rectangular playing area for children. The fencing for the front and back sides costs $10 per meter and the fencing for the other two sides costs $15 per meter. Find the dimensions of the largest possible playing garden.

**Solution:**
maximize \( f(x,y) = xy \) , with subject to \( g(x,y) = 2(10x) + 2(15y) = 3000 \)

\[ \frac{f_x}{g_x} = \frac{f_y}{g_y} \iff \frac{y}{20} = \frac{x}{30} \iff 2x = 3y \]
Plug this in the constraint \( 20x + 30y = 3000 \), we get \( x = 75 \) and \( y = 50 \)

**Problem 3: (least square method) ( 3 points)**
Find the equation of the line that best fits the following data, given as a collection of points: \((1,2) ; (2,3) ; (3,5) ; (4,9) ; (5,11) ; (6,12) ; (7,14)\).

Write the value of \( m \) and \( b \) (of the equation \( y = mx + b \)) in rational format (like \( \frac{1}{2}, \frac{4}{7} \) instead of 0.5, 0.5714... ).

<table>
<thead>
<tr>
<th>( x_i )</th>
<th>( y_i )</th>
<th>( x_i - \hat{x} )</th>
<th>( y_i - \hat{y} )</th>
<th>( (x_i - \hat{x})(y_i - \hat{y}) )</th>
<th>( (x_i - \hat{x})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-3</td>
<td>-6</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-2</td>
<td>-5</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>-1</td>
<td>-3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

mth122spring2009
Therefore, \( m = \frac{60}{28} = \frac{15}{7} \) and \( b = 8 - \frac{15}{7} \cdot 4 = -\frac{4}{7} \).

The equation of the best line is \( y = \frac{15}{7} x - \frac{4}{7} \).