**Practice homework 2**

**Problem 1:**
Apply optimization method and find (if possible) the value of x and y that optimize the value of the given function. What is the optimal value?

[a] \( f(x, y) = 5x^2 + 16xy + 14y^2 - 4x - 4y + 20 \)

[b] \( f(x, y) = x^2 - 2y^2 + 2x + 4y - 5 \)

[c] \( f(x, y) = x^2 - 4xy + 6y^2 + 2x + 12 \)

[d] \( f(x, y) = 5x^2 + 2y^2 - 6xy + 2x - 2y + 1 \)

**Problem 2:** (*)
Find the minimum value of the following function. Which are the values of x and y that make f achieve this optimal result?

\[ f(x, y) = x^4 + 2y^2 - 2x^2y - 4y + 5 \]

(hint: \( f_x = 4x^3 - 4xy = 4x(x^2 - y) \). This is equal to zero only if \( x = 0 \) or \( y = x^2 \))

(hint: do not be afraid)
Solution:

Problem 1:

[a] \( f_x = 10x + 16y - 4 \), \( f_y = 16x + 28y - 4 \), \( f_{xx} = 10 \), \( f_{yy} = 28 \), \( f_{xy} = 16 \)

\[ D = (10)(28) - 16^2 = 24 > 0 \], \( f_{xx} > 0 \), therefore \( f \) has a (local) minimum.

Solve \( f_x = f_y = 0 \) we get \( x = 2, y = -1 \) and minimum \( f \) is \( f(2,-1) = 18 \).

[b] \( D = (2)(-4) - 0^2 < 0 \), no optimal solution.

[c] There is a minimal solution \( x = -3, y = -1, f(-3,-1) = 9 \)

[d] The optimal solution is \( x = 1, y = 2, f(1,2) = 0 \).

Problem 2:

\( f_x = 4x^3 - 4xy \), \( f_y = 4y - 2x^2 - 4 \)

Like in the hint, when we solve \( f_x = 0 \) we have either \( x = 0 \) or \( y = x^2 \).

If \( x = 0 \), solve \( f_y = 4y - 4 = 0 \) we get \( y = 1 \).

If \( y = x^2 \) then the 2\(^{nd} \) equation becomes \( 4x^2 - 2x^2 - 4 = 0 \),
which means \( x^2 = 2 \) and \( x = \pm \sqrt{2} \) and \( y = 2 \).

To check those candidates, we compute

\( f_{xx} = 12x^2 - 4y \), \( f_{yy} = 4 \), \( f_{xy} = -4x \)

For \( x = 0, y = 1 \), we see that \( D = (-4)(4) - 0 = -16 \). Not a solution.

For \( x = \pm \sqrt{2}, y = 2 \), we see that \( D = (16)(4) - 32 = 32 > 0 \).

Conclusion, there are 2 solutions \( (x = \sqrt{2}, y = 2) \) and \( (x = -\sqrt{2}, y = 2) \), both gives the minimum value \( f = 1 \).