Homework 2
due on 1st class of week 8

Problem 1 (3 points)
Compute derivatives of the following functions using quotient rule:

[a] \( f(x) = \frac{x}{2x+5} \)
[b] \( f(x) = \frac{5x+7}{2-x} \)
[c] \( f(x) = \frac{x+1}{x^2+1} \)

Solution:
[a] \( u = x, u' = 1; v = 2x+5, v' = 2; \)
\[ f'(x) = \frac{(1)(2x+5)-(x)(2)}{(2x+5)^2} = \frac{5}{(2x+5)^2} \]

[b] \( u = 5x+7, u' = 5; v = 2-x, v' = -1; \)
\[ f'(x) = \frac{(5)(2-x)-(5x+7)(-1)}{(2-x)^2} = \frac{17}{(2-x)^2} \]

[c] \( u = x+1, u' = 1; v = x^2+1, v' = 2x; \)
\[ f'(x) = \frac{(1)(x^2+1)-(x+1)(2x)}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2} \]

Problem 2 (3 points)
Compute derivatives of the following functions using chain rule:

[a] \( f(x) = (x^2+5)^8 \)
[b] \( f(x) = \sqrt{x^4+10x^2+16} \)

Solution:
[a] \( f'(x) = 8(x^2+5)^7(2x) = 16x(x^2+5)^7 \)

[b] \( f'(x) = \frac{4x^3+20x}{2\sqrt{x^4+10x^2+16}} = \frac{2x^3+10x}{\sqrt{x^4+10x^2+16}} \)

Problem 3 (4 points)
Given the following relation: \( x^3+y^2-4=x^2+y \) (almost identical to the example in lecture)

[a] Compute implicit derivative \( \frac{dy}{dx} \) (or \( y' \)).

[b] Check that \( x = 2, y = 1 \) satisfy the relation

[c] Evaluate \( y' \) at \( x = 2, y = 1 \) (just plug these values in the answer you get for [a]).

[d] Find the tangent line of the curve (i.e. graph of this relation) at \( x = 2, y = 1 \) (hint: you know the slope from [c], and you know the point, use point-slope equation).

Solution:
[a] Differentiate both sides: \( 3x^2+2yy' = 2x+y' \), which implies \( 2y-1)y' = 2x-3x^2 \)
This yields: \( y' = \frac{2x-3x^2}{2y-1} \)

[b] Replace \( x \) by 2, \( y \) by 1: \( (2)^3+(1)^2-4=(2)^2+(1) \). CORRECT!

[c] From [a], replace \( x = 2, y = 1 \), the result is \( y' = -8 \)

[d] tangent line: \( y = -8(x-2) + 1 = -8x + 17 \)