Chapter 1  
Functions, Graphs and Limits

**Functions**  
A function is a rule that assigns to each object in a set A exactly one object in a set B. The set A is called the domain of the function and the set of assigned objects in B is called the range.

Notations: \( f(x) \) or \( y = f(x) \)

Composition of functions (\( f \circ g \)): \( f(g(x)) \) combine 2 or more functions in a chained order.

Example: in lecture

**Topics:** determine the domain and the range of a function

Domain of a function \( f \): the set of all values of \( x \) for which \( f(x) \) is defined (can be computed)

*Hint:* exclude values that lead to dividing zero or taking square root of a negative quantity.

Range of a function \( f \): all the values that can be presented as the output of \( f \) corresponding to some input \( x \)

**Topics:** functional model  
Given a relation between 2 quantities \( x \) and \( y \), find out the function \( f \) such that: \( y = f(x) \).

*explicit problem*: the relation is interpreted straight forward.
  
  For example: compute the circumstance of a circle, knowing its radius.
  
  *profit function.*

*implicit problem*: need to do some algebra to find out the function.

**Graph of a function**  
Present the function geometrically in a rectangular coordinate system, as a collection of all points \((x,y)\) where \( x \) is in the domain of \( f \) and \( y = f(x) \). In other words, all the points \((x,f(x))\)

By looking at the graph, one can tell if the function is continuous, piece-wised continuous, differentiable, ...

**Topics:**  
Find the \( x \) and \( y \) intercepts

*\( x \_intercept \)*: the point where the graph hits the \( x \) axis

*\( y \_intercept \)*: the point(s) where the graph hits the \( y \) axis

To compute \( y \_intercept \): put \( x = 0 \), compute \( y \).

To compute \( x \_intercept \): put \( y = 0 \), solve for \( x \). (equation solving problem)

**Linear Functions**  
Function formula: \( f(x) = mx + b \) or \( y = mx + b \) where \( m \) and \( b \) are two constants.

The graph of a linear function is a straight line and \( m \) is also called *slope*

Note: the \( y \_intercept \) is \((0,b)\)

**Topics:** linear equations  
solve equation \( mx + b = 0 \)

Form \( y = mx + b \) can be implied as slope-intercept form of equation of a line

**Topics:** the slope of a line

\[
\text{slope} = \frac{\text{change in } x}{\text{change in } y} = \frac{\Delta x}{\Delta y} = \frac{y_2 - y_1}{x_2 - x_1}
\]

The point-slope form of the equation of a line: \( y - y_0 = m(x - x_0) \)

**Topics:** equation of a line going through 2 separate points \((x_1,y_1)\) and \((x_2,y_2)\)
Topics: Parallel and Perpendicular Lines

\( (L_1) \ y = m_1 x + b_1 \) and \( (L_2) \ y = m_2 x + b_2 \)

\( L_1 \) and \( L_2 \) are parallel iff \( m_1 = m_2 \)

\( L_1 \) and \( L_2 \) are perpendicular iff \( m_1 = \frac{-1}{m_2} \)

### Quadratic Equations

solve the equation: \( a x^2 + b x + c = 0 \) where \( a, b, c \) are constants

**discriminant** \( \Delta = b^2 - 4ac \)

if \( \Delta < 0 \) equation has no solutions.

if \( \Delta = 0 \) equation has one solution \( x = \frac{-b}{2a} \)

if \( \Delta > 0 \) equation has 2 distinct solutions: \( x_1 = \frac{-b - \sqrt{\Delta}}{2a} \) and \( x_2 = \frac{-b + \sqrt{\Delta}}{2a} \)

Graph of a quadratic function \( a x^2 + b x + c \) is a parabola