Chapter 3
Applications of The Derivative

**Increasing and Decreasing**
Let $f(x)$ be a function defined on the interval $a < x < b$.

If $f'(x) > 0$ for all values of $x$ in the interval, then $f$ is increasing on the interval.

If $f'(x) < 0$ for all values of $x$ in the interval, then $f$ is decreasing on the interval.

**Topics: determine the intervals of increase and decrease**
(refer to lecture for completed examples)
- Compute $f''$.
- Find all values of $x$ for which $f'$ equals zero or $f''$ is not defined.
- List all these values on the first row of a chart-table. List all the terms on the first column.
- Determine the $[+]$ and $[-]$ signs on each intervals.
- Determine the sign of $f''$ by counting the number of $[-]$ signs on each interval. An odd number of $[-]$ signs yields a $[-]$. An even number of $[-]$ signs yield a $[+]$.

Note: this method is slightly different from the one presented in textbook

**Local maximum and local minimum**
A function $f$ has a local maximum (minimum) at $x = c$ if $f(z) \leq f(c)$ ($f(z) \geq f(c)$, respectively) for all values of $z$ in a neighborhood of $c$.

**Topics: find local max and local min**
First, compute $f'$. If the sign of $f'$ changes from $[-]$ to $[+]$, it's a local maximum; similarly, if the sign changes from $[+]$ to $[-]$, it's a local minimum.

**Topics: find global max and global min**
To find global maximum, we only have to find all the local max, then compare with the values of $f$ on the boundary, take the largest. Similar for global minimum.

**Concavity**
Given a function $f$, 2nd derivative $f''$.

If $f''(x) > 0$ for all values of $x$ in the interval $a < x < b$ then $f$ is concave up on the interval.

If $f''(x) < 0$ for all values of $x$ in the interval $a < x < b$ then $f$ is concave down on the interval.

**Topics: optimization**
Model the objective quantity as a function $f(x)$ of some variable $x$. Determine the suitable range of the values of $x$. Solve the optimization problem by finding the global maximum or minimum of the function $f(x)$.