Chapter 7
Function of several variables

Function of two variables
A function $f$ of two independent variables $x$ and $y$ is a rule that assigns each ordered pair $(x, y)$ in a given domain $D$ exactly one value, denoted by $f(x, y)$.
Many examples in lectures.

Partial derivative
Take derivative of the function $f(x, y)$ with respect to $x$ or $y$.
When differentiating the function in $x$, we regard $y$ as a constant.
Notation: $f_x = \frac{\delta f}{\delta x}$, $f_y = \frac{\delta f}{\delta y}$, $f_{xy} = \frac{\delta^2 f}{\delta y \delta x}$, ...

Double integration
Integrate a function $f$ with respect to one variable and fix the others, we get another function of fewer variable, then we integrate the new function.
$$\int \int f(x, y) \, dy \, dx = \int \left[ \int f(x, y) \, dy \right] \, dx \quad \text{and} \quad \int \int f(x, y) \, dx \, dy = \int \left[ \int f(x, y) \, dx \right] \, dy$$

Theorem:
If $f$ is a continuous function then:
$$\int_a^b \int_c^d f(x, y) \, dx \, dy = \int_c^d \int_a^b f(x, y) \, dx \, dy$$
If $R$ is the rectangular $a \leq y \leq b$, $c \leq x \leq d$ then we can write the above integration as:
$$\int_R f \, dA$$

Topics: Volume
If $f(x, y)$ is continuous and positive on a rectangular region $R$ then the solid region under the surface $z = f(x, y)$ over the region $R$ has volume given by
$$\int \int_R f \, dA$$