mth131 Fall 2008

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Test One
September 22 2008

Your name:
Student #:

Important:
✓ Show your work. A sole answer without anything to show that you honestly solve the problem will be graded zero.
✓ You can use a calculator, but only to help you in calculating or checking your work. Simply copying answer from calculator will be considered cheating (for example, input a quadratic equation in a scientific calculator and copy its solution).
✓ You can use a print of the online lecture note, NOT YOURS, so it can be considered fair for all students. Along with the note, you can also bring a small cheat-sheet (not a A4 sized one).
✓ I DO give partial credits, at my standard. This means, writing some weird stuff and claim it solved the question doesn’t mean you will get some credits.
✓ I prefer you give answer in rational value or radical form, a decimal answer (like, 2.6458 in stead of \( \sqrt{7} \)) is ugly and in some situation will not be acceptable.
✓ No make-up test !!! except super extraordinary reason.
✓ Don’t forget to write your name on the first page of your work, or on this sheet and staple to your papers. Also remember to write your name on the back of the last sheet which helps me in returning your tests later.

Test questions:

Problem 1: 5 points
Find the domain of the following function:

\[ f(x) = \frac{1}{x-2} + \sqrt{3-x} \]

Solution:
We need \( x-2 \neq 0 \) and \( 3-x \geq 0 \), which means \( x \leq 3 \) and \( x \neq 2 \)

Problem 2: 5 points
It costs a book publisher $60,000 to prepare a book for publication (typesetting, illustrating, editing and so on); printing and binding costs are $4 per book. The book is then sold to bookstores for $20 per copy. Write expressions representing:
[a] (1 point) the cost C(x)
[b] (1 point) the revenue R(x)
[c] (1 point) the profit P(x)
[d] (2 points) How many books should the publisher sell to start making profit?

Solution:
[a] \( C(x) = 60,000 + 4x \)
[b] \( R(x) = 20x \)
[c] \( P(x) = R(x) - C(x) = 16x - 60,000 \)
[d] \( P(x) \geq 0 \iff x \geq 60,000 / 16 = 3750 \) (copies)
Problem 3: 5 points
You are given a quadratic function: \( f(x) = x^2 + 8x - 15 \) and its graph is a parabola (P).

[a] (1 point) Find the \( y \)-intercept of P and y axis
[b] (2 points) Find the two \( x \)-intercepts of P and x axis (that means, solve \( f(x) = 0 \))
[c] (2 points) Find the equation of the line which goes through the vertex of P and is perpendicular to the line \( y = 4x \)

Solution:
[a] \( y \)-intercept : \( (0, -15) \)
[b] \( \Delta = 8^2 - 4(1)(-15) = 64 + 60 = 124 \), therefore \( x_{1,2} = \frac{-8 \pm \sqrt{124}}{2} = -4 \pm \sqrt{31} \)
Hence, \( x \)-intercepts are \( (-4 + \sqrt{31}, 0) \) and \( (-4 - \sqrt{31}, 0) \)
[c] \( x = -8/2 = -4, y = (-4)^2 + 8(-4) - 15 = -31 \)
Therefore we have \( y = \frac{-1}{4}(x + 4) - 31 = -\frac{1}{4}x - 32 \)

Problem 4: 5 points
Given a function: \( f(x) = \frac{(3x^2 + 2)(x - 4)}{(x^2 - 16)(x - 2)} \). Compute the following limits:
[a] (1 point) \( \lim_{x \to \infty} f(x) \)
[b] (1 point) \( \lim_{x \to 0} f(x) \)
[c] (3 points) \( \lim_{x \to 4} f(x) \)

Solution:
[a] \( \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{3x^2(x)}{(x^2)(x)} = 3 \)
[b] \( \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(2)(-4)}{(-16)(-2)} = \frac{-1}{4} \)
[c] \( \lim_{x \to 4} f(x) = \lim_{x \to 4} \frac{(3x^2 + 2)(x - 4)}{(x - 4)(x + 4)(x - 2)} = \lim_{x \to 4} \frac{3x^2 + 2}{(x + 4)(x - 2)} = \frac{3(4^2) + 2}{(4 + 4)(4 - 2)} = \frac{50}{16} = 8 \)

Problem 5: 10 points
Use Least Square method to find the equation of the line that best fits the following data:
\( (1, 2); (2, 3); (3, 5); (4, 9); (5, 11) \)

Solution: \( \hat{x} = 3, \hat{y} = 6, m = 24/10, b = -12/10 \) and the line is \( y = 2.4x - 1.2 \)

\[
\begin{array}{cccccc}
x_i & y_i & x_i - \hat{x} & y_i - \hat{y} & (x_i - \hat{x})(y_i - \hat{y}) & (x_i - \hat{x})^2 \\
1 & 2 & -2 & -4 & 8 & 4 \\
2 & 3 & -1 & -3 & 3 & 1 \\
3 & 5 & 0 & -1 & 0 & 0 \\
4 & 9 & 1 & 3 & 3 & 1 \\
5 & 11 & 2 & 5 & 10 & 4 \\
\end{array}
\]
Problem 6: 5 points
Compute the derivative of the following function:

\[ f(x) = \frac{x^3 + 1}{x} + 5x \]

Solution:
\[ f(x) = x^2 + x^{-1} + 5x \quad \text{therefore} \quad f'(x) = 2x - x^{-2} + 5 \]

Bonus question: 5 points
Find the equation of the line which is tangent to the graph of \( y = f(x) \) at \( x = 1 \)

Solution:
\[ x_0 = 1, \quad f(x_0) = f(1) = 1 + 1 + 5 = 7, \quad f'(x_0) = f'(1) = 2 - 1 + 5 = 6 \]
Equation of tangent line:
\[ y = 6(x - 1) + 7 = 6x + 1 \]

Problem 7: 5 points
Find the derivative of:

\[ f(x) = (x^3 - 9x + 1)(5x^4 + 3x - 2) \]

Solution:
\[ f'(x) = (3x^2 - 9)(5x^4 + 3x - 2) + (x^3 - 9x + 1)(20x^3 + 3) \]

Extra credit question: 5 points
Approximate: \( Q = \frac{1}{1.004^{10}} \). Show details.

Solution:
\[ f(x) = x^{-10}, \quad x = 1, \quad h = 0.004, \quad f'(1) = 1, \quad \text{also} \quad f'(x) = -10x^{-11}, \quad f'(1) = -10 \]
Therefore, \( Q = f(x + h) \approx f(x) + hf'(x) = 1 + 0.004(-10) = 1 - 0.04 = 0.96 \)